L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2026 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	2^{nd} May 2026
TIME:	9:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are **not** allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.

1. (a) Solve for $x > 0$ the equation

$$\log_7(2x+5) - \log_7(x-5) = \log_7\left(\frac{x}{2}\right).$$

[4 marks]

(b) Simplify

$$\frac{\sin\vartheta}{1+\cos\vartheta} + \frac{1+\cos\vartheta}{\sin\vartheta}.$$

[3 marks]

[**Total: 7 marks**]

2. The acceleration of a body depends on time t and velocity v according to the equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = v\,\sin 1.9t\,,$$

where the argument of the sine function is in radians. Given that when $t = 0$ seconds, the body has a velocity of $1\ ms^{-1}$, show that

$$\ln|v| = -0.53\,\cos 1.9t + 0.53\,.$$

Hence, find the velocity of the body when $t = 6\ s$. Give your answer correct to two decimal places.

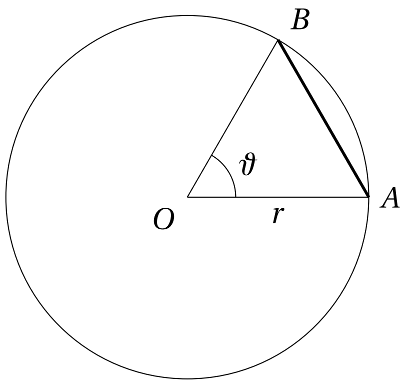
[5, 2 marks]

[**Total: 7 marks**]

3. (a) Assuming that the function $f(x) = \dfrac{\sin x}{x}$ is one-to-one on $(0, \pi)$, explain why the equation $2\pi \sin x = 3\sqrt{3}x$ has a unique solution $x = \pi/3$ in the interval $(0, \pi)$.

[3 marks]

(b)



The figure shows a circle with centre O and radius r. The chord AB subtends an angle ϑ radians at O.
Given that the ratio of the area of the minor segment of AB to that of the sector AOB is $1 - \frac{3\sqrt{3}}{2\pi}$, find ϑ. Hence, find the perimeter of the minor segment, giving your answer in terms of r.

[4, 3 marks]

[**Total: 10 marks**]

4. (a) Sketch the curves $y = x^2 - \frac{\pi^2}{16}$ and $y = \cos(2x)$ on the same set of axes for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

[5 marks]

(b) Find the area enclosed by the two curves.

[5 marks]

[**Total: 10 marks**]

5. Three matrices are given by $A = \begin{pmatrix} 2 & x \\ k & 2 \end{pmatrix}$, $B = \begin{pmatrix} x^2 & 1 \\ 5 & 6 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}$, where k and x are real numbers.

(a) Given that the determinant of A is equal to the determinant of B, show that

$$6x^2 + kx - 9 = 0. \qquad (\dagger)$$

[2 marks]

(b) Show that for every value of k, there are **TWO** distinct values of x satisfying equation (†).

[2 marks]

(c) Let $k = 3$. Find the value of x, given that $x > 0$. Taking this value of x, find the matrix D such that $D = AC$. Describe the transformation represented by matrix D.

[2, 2, 2 marks]

[**Total: 10 marks**]

6. (a) The third term of an arithmetic progression is 10, while the sixth term is 19.
   (i) Find the first term and the common difference.
   (ii) Find the sum of the first 20 terms of this arithmetic progression.

[2, 2 marks]

   (b) The first term of a geometric progression is 4 and the fourth term is 1372.
   (i) Find the common ratio and the value of the second term of this geometric progression.
   (ii) The second term of the geometric progression is equal to the m'th term of the arithmetic progression in (a). Find m.

[2, 2 marks]

   (c) What is the coefficient of x^3 in the expansion of $(2x+9)^5$?

[2 marks]

[**Total: 10 marks**]

7. (a) A box contains 6 red and 8 yellow beads. Two beads are selected at random from the box, one after the other, without replacement. By using a tree diagram, or otherwise, find the probability that the two selected beads have a different colour.

[4 marks]

   (b) Find the number of arrangements of all the letters of the word APPARATUS. How many of these arrangements:
   (i) start and end in P? (ii) have the A's next to each other?

[2, 2, 2 marks]

[**Total: 10 marks**]

8. The line ℓ_1 passes through the points $A(1,3)$ and $B(6,8)$.
   (a) Find the equation of ℓ_1. Find also the distance between the points A and B, leaving your answer in the simplest surd form.

[4 marks]

   The line ℓ_2 passes through A and is perpendicular to the line ℓ_1.
   (b) Find the equation of ℓ_2.

[2 marks]

   (c) Draw the two lines on the same set of axes, showing clearly the x- and y-intercepts.

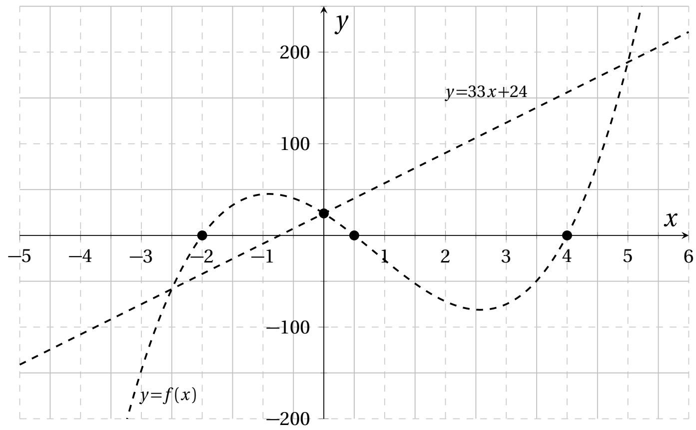
[3 marks]

   (d) Find the area enclosed by the two lines and the two axes.

[4 marks]

[**Total: 13 marks**]

9. The diagram below shows the graphs of $y = g(x)$ and $y = f(x)$, where $g(x) = 33x + 24$ and $f(x) = k(x-a)(x-b)(x-c)$.



(a) Use the graph to determine the values of a, b, c and k.

[3 marks]

(b) Solve the equation $f(x) = g(x)$.

[3 marks]

(c) Find the values of A, B and C such that

$$\frac{x+1}{f(x)} \equiv \frac{A}{3(x+2)} + \frac{B}{2x-1} + \frac{C}{x-4}.$$

[4 marks]

[**Total: 10 marks**]

10. A light source is placed at point P with coordinates (x, y) on the curve $y = 4 + \ln(x+1)$ for $x \geq 0$. A sensor is located at the origin O.

(a) Find the gradient of the curve at the point x.

[2 marks]

(b) A robotic arm moves the light source along the curve such that the x-coordinate increases at a constant rate of 0.5 units per second. Find the rate of change of the y-coordinate when $x = 3$.

[3 marks]

(c) The efficiency E of the light reaching the origin is modeled by the formula

$$E(x) = \frac{\ln\sqrt{x+1}}{x+1}.$$

(i) Show that

$$E'(x) = \frac{1-\ln(x+1)}{2(x+1)^2}.$$

(ii) Deduce that the efficiency E has a stationary point. Determine its nature.

[4, 2, 2 marks]

[**Total: 13 marks**]