UNIVERSITY OF MALTA

THE MATRICULATION CERTIFICATE EXAMINATION
ADVANCED LEVEL

PURE MATHEMATICS

May 2010

EXAMINERS' REPORT

MATRICULATION AND SECONDARY EDUCATION
CERTIFICATE EXAMINATIONS BOARD

Summary of Results

Grade	A	B	C	D	E	F	Abs	Total
Number	31	83	134	49	84	233	41	655
%	4.7	12.7	20.5	7.5	12.8	35.6	6.2	100

Summary of Lower Marks

≤	10	15	20
Number	65	96	141
%	9.9	14.6	21.5

Comments on Candidates' Performance

Paper 1

Question 1. Two main correct methods were used by the majority of the candidates in answering part (a) of this question: (i) the completing the square method; and (ii) finding the values of a, p and q by equating coefficients. The sketch of the graph in (b) was generally well attempted, but a considerable number of candidates were unable to correctly solve the given inequality. In part (c), the inequality involving the discriminant constructed by many candidates was inadequate, and some of those constructing the correct inequality were still unable to appropriately solve it.

Question 2. The majority of the candidates identified correctly the differential equation as "separable variables" type. However there was quite a significant number of candidates who were unable to separate the variables correctly. This involved using trivial rules of algebra and lack of such knowledge is not expected at this level. As regards those who managed to do the first part correctly, it was then simple for them to continue the problem since it involved integration of standard functions.

Question 3. (a) The great majority of the candidates either inappropriately tried to use inverse matrices, or simply made up a couple of matrices $\mathbf{Q}$ satisfying the given equation. However, the few candidates who adopted a proper approach by writing $\mathbf{Q}$ as a general 2×2 matrix were successful in subsequently finding all the required matrices $\mathbf{Q}$. (b) Although there were a few candidates who did not even know from where to start answering this question, in general the images of the given vertices were correctly determined. The determinant was correctly evaluated by the majority, but many of the different methods used to find the area of S' were either completely wrong or contained several mistakes.

Question 4. Despite the fact that many candidates showed lack of knowledge of the basic principles of vectors, the vast majority of the students managed to obtain quite a high mark. It was also quite interesting how a substantial number of candidates managed to solve this problem using Pythagoras Theorem and simple trigonometry, thus avoiding completely the use of the dot product. In part (c), although it was a straight forward question, many candidates simply applied the formula to find the perpendicular distance of a point from a straight line

$$\left|\frac{ah+bk+c}{\sqrt{a^2+b^2}}\right|$$

which is used in coordinate geometry without realising that it cannot be used directly in this case.

Question 5. (a) Some candidates first expressed z in polar form and then used De Moivre's Theorem to find $\sqrt{z}$ and $\frac{1}{\sqrt{z}}$. However, in doing so, many gave only one value for each, instead of the required two. Those who started correctly by using the Cartesian form were more successful in finding the required complex numbers. (b) The majority of the candidates correctly used the factor theorem to prove the given statement, and then appropriately deduced that the complex conjugate is also a root. However, there were fewer candidates who correctly found the third root of the equation. An alternative approach adopted by some candidates was to factorise the given cubic expression directly, and hence immediately finding the three roots.

Question 6. Most of the candidates answered correctly parts (a) and (b) of this question, however there was not a single candidate who gave the correct answer for the domain of the composite function $f \circ g$. All the candidates who answered this part of this question just considered the resulting composite function forgetting about the separate domains of f and g. A significant number of candidates managed to answer part (c) correctly as well, but obviously giving the incorrect range of the inverse of h since its domain was not correct in the previous part. Also most of the sketches were too messy, most of the times without indicating properly the position of the asymptotes and the intercept.

Question 7. (a) Many candidates correctly answered this question. However, among those who did not answer correctly, some did not consider order to be important, some forgot to consider the rotational symmetry of the ring, and others did not divide by two to take into account the flipping of the ring. (b) The first and last parts of this question were, in general, correctly answered, but many candidates did not answer the second part correctly. Some incorrectly subtracted the answer of the last part from that of the first, and thus ignoring the cases where only two of the **O**'s come together.

Question 8. The vast majority of the candidates got the first part of this question correct. In the second part of section (a), a significant number of candidates had no idea how to tackle the problem, but even those who managed to start it, a significant number just found the maximum and minimum value of $2\cos 3x + 4\sin 3x + 13$ while they were asked to find these values for the reciprocal of that function. Regardless of what they did in part (a), most of the candidates started correctly part (b), but a significant number of them just found the principle value and wrote the general solution in the generic form, without simplifying and thus arriving to the final answer.

Question 9. This question was very poorly answered. Some candidates assumed particular values of q and tried to solve the equation for these values. There were also a considerable number of candidates who incorrectly treated the given cubic equation as a quadratic equation, and then considered the discriminant in an attempt to find the number of roots and their nature.

Question 10. Most candidates found it more difficult to solve the partial fractions rather than the integral itself in part (a), however they were still able to integrate the given function. There was a significant number of candidates who obtained a "zero" for either A or B, thus oversimplifying the integral that followed. In part (b), most of the candidates did very well, with few exceptions where they chose the incorrect functions, which to differentiate and which to integrate, some integrated incorrectly e^{4x} as $4e^{4x}$ and some even as $5e^5x$. A significant number of candidates got part (c) correct but there were also many candidates who did not even manage to choose the correct trigonometric identity needed to simplify the integrand, which was quite elementary, thus making it practically impossible to complete the integration.

Paper 2

Question 1. Many candidates attempted this question and many obtained full marks. However some candidates found difficulty in working out the integrating factor and others in finding the second derivative of Pxe^{3x}. Errors were also made in solving the auxiliary equation giving the roots as 6 and 1 instead of 2 and 3.

Question 2. Few candidates attempted this question (28%) with appalling results. The problem seems to be a lack of a methodological approach towards understanding the underlying situation and hence applying the right tools. Combinatorial problems tend to be answered in unexplained one or two lines. There were also many cases where candidates quoted numbers greater than one as probabilities.

Question 3. Many of the candidates who attempted this question worked out correctly the position vector of a point where two lines meet, the equation of a plane that contains two lines and the angle between two planes. Finding the position vector of a point where the line crossed a plane and the equation of a plane perpendicular to two plane was beyond many candidates.

Question 4. A very popular question, attempted by 83% of the candidates. Part (a) was very well tackled, candidates producing very good sketches, although some students still think that in polar coordinates the equation $r = 2$ represents a straight line. A few still find difficulties in finding the correct limits of integration for the area. A significant number left out part (b) of the question maybe because this was the first time the students were asked to find the volume of a solid of revolution in polar coordinates although the formula for the volume was given. The method for finding the volume was the same as that of finding the area except that the integrand was $r^3 \sin\theta$. Some candidates integrated r^3. In finding

$$\int (2 + \cos\theta)^3 \sin\theta \ \mathrm{d}\theta\,,$$

most candidates went to all the trouble of expanding $(2+\cos\theta)^3$ then made the necessary substitution instead of substituting directly to obtain $-\frac{1}{4}(2+\cos\theta)^4$.

Question 5. Candidates knew how to use the Newton-Raphson method but some found it difficult in differentiating $x^2 = \arctan x + 1$. In working out part (b) of the question, mistakes were done in the formula and in differentiating $y = \sqrt{x^2 - 1}$. Candidates were familiar with Simpson's rule.

Question 6. Attempted by only 28% with a very low average mark of 1.5. Only one candidate managed to score full marks. This was a straight forward question based on the simple hyperbolic identity $\cosh^2 x - \sinh^2 x = 1$. Very few managed to show that $\cosh x = \sec\theta$, $\tanh x = \sin\theta$ and $\dfrac{\mathrm{d}\theta}{\mathrm{d}x} = \cos\theta$. The evaluation of the two integrals was very disappointing. Most candidates did not bother to make the necessary change of variable. Part (b) was also based on the above identity. Only a few arranged the formula in the form

$$I_{n+1} + I_n = \int_0^t \frac{\sinh^{2n} x(1+\sinh^2 x)}{\cosh x}\,\mathrm{d}x\,.$$

Most candidates used integration by parts without any success. Only one candidate evaluated $I_0 = \int_0^1 \frac{1}{\cosh x}\,\mathrm{d}x$ correctly.

Question 7. This question which was attempted by 52% was generally answered in an unsatisfactorily way. Part(a) was fairly well answered. When proving the result for $n = 1$, many of the solutions given were

$$\frac{x^2-1}{x-1} = 1 + x + x^2 + \cdots + x^n + x^1\,.$$

In proving the result for $n = k+1$, most candidates simply quoted that

$$\frac{x^{k+2}-1}{x-1} = \frac{xx^{k+1}-1}{x-1} = 1 + x + x^2 + \cdots + x^{k+1}\,,$$

without showing any working. A few number of candidates found this part very easy by simply writing

$$\sum_{r=0}^{k+1} x^r = \sum_{r=0}^{k} x^r + x^{k+1}\,.$$

Proving the inequalities $n^2 \geq 2n+1$ and $2^n \geq n^2$ showed very clearly that the understanding of the induction principle is extremely shaky. Many of the candidates assumed that the statement is true for $n = k+1$, then went on to prove some other result such as $k^2 > 0$.

Question 8. Fairly well attempted by 75%. Many candidates found difficulties in finding the partial fractions of the given expression which could have been obtained simply by the cover up rule. For the sum of the given series almost all candidates found the sum to n terms by summing from $r = 1$ to n instead of summing from $r = 2$ to $n+1$. In part (b), most candidates did not seem to know how to manipulate expressions involving factorial numbers. Only a few managed to prove the result. Many attempted to write

$$\frac{n(3+2n)}{(n+2)!} \quad \text{as} \quad \frac{n(3+2n)}{(n+1)(n+2)n!} = \frac{A}{n!} + \frac{B}{(n+1)!} + \frac{C}{(n+2)!}\,,$$

but, did not obtain the required constants. The last part was very poorly attempted. The majority of the students did not know how to relate

$$\sum_{n=1}^{\infty} \frac{2}{n!} - \sum_{n=1}^{\infty} \frac{3}{(n+1)!} + \sum_{n=1}^{\infty} \frac{2}{(n+2)!}$$

to the expansion of e^x.

Question 9. Some candidates were unable to work out correctly the augmented matrix. Although some knew what were the conditions to have a unique solution, no solution, or an infinite number of solutions, they were unable to find the solutions where possible. Nearly no one got the second part of the question correct.

Question 10. A few candidates tried to solve without success the first part of the question using the identity $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$. In part (b) of the question few realized that they had to multiply the numerator and the denominator by $x - i(y+b)$.