

EXAMINERS' REPORT

Pure Mathematics
Advanced Matriculation Level

First Session 2019



**L-Università
ta' Malta**

**MATSEC
Examinations Board**

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Summary of Results

Grade	A	B	C	D	E	F	Abs	Total
Number	38	87	140	38	33	109	55	500
%	7.6	17.4	28.0	7.6	6.6	21.8	11.0	100

Summary of Lower Marks

\leq	10	15	20
Number	37	49	58
%	7.4	9.8	11.6

Comments on Candidates' Performance

Paper 1

- Question 1. Candidates performed quite well in this question. Some candidates found difficulty in integrating $e^{-y} \cos y$, mostly due to:
- (a) silly mistakes in signs on performing substitutions, or
 - (b) the fact that the integrals on the LHS and RHS were worked out simultaneously, thus candidates could not collect the same integral on one side and then divide by 2.
- Question 2. Candidates performed well in this question, showing a good knowledge of differentiation. When proving the result in (c), candidates were expected to either rearrange the result for the second derivative to reach the given equation, or substitute for y , y' and y'' in the LHS of the given equation to show it is equal to 2. However, in some cases, students were simplifying the given equation and moving terms from RHS to LHS or vice-versa until they reached two equal terms on either side.
- Question 3. 41% of candidates got 5 marks or more in this question. Part (a) was straight forward and the vast majority gave the correct answer although there was a common misconception of using vector difference instead of sum to find the mid-point. Although most candidates appeared to know what is required to answer parts (b) and (c), a significant number of them did a lot of mistakes in the working, thus rendering the rest of the solution very difficult to follow.

- Question 4. (a) Although a significant number of candidates got the right expressions for $f \circ g$ and $g \circ f$, more than half of them got at least one of them wrong and thus compromising the rest of the solution for part (a). One can also note that the vast majority of candidates who got both expressions right showed clearly they do not understand the fact that two functions represented by the same expression but on different domains are different functions.
- (b) Although the vast majority of candidates started their sketch using as a model that of $y = \ln x$, very few were able to give the required sketch. It is worth mentioning that many candidates performed the proper transformations on x instead of $-x$ and thus obtained an incorrect sketch. Most of them managed to get the expression for the inverse function correct but gave the incorrect domain. In most cases this was due to their incorrect sketch.
- Question 5. (a) Most candidates found this part very challenging. Although a significant number of them managed to solve the first part of (a), they did not manage to follow through and obtain a correct solution.
- (b) Most candidates managed to obtain $\alpha + \beta$ and $\alpha\beta$, and a significant number of them managed to continue and obtain a solution, apart from some numerical mistakes. In fact 54% got 5 marks or more in this question.
- Question 6. In part (a), a good number of candidates ignored the given hint and simply cross multiplied by x , without stating whether x was taken to be positive or negative, leading to one inequality which was solved graphically. Others first multiplied by x and then tried to (unsuccessfully) introduce the given cases at a later stage. Most of those who did consider the two suggested cases then forgot to impose the restriction $x > 0$ or $x < 0$ on the range of values they obtained from their cubic curve/s. It was also noted that in quite a number of cases, when factorising the resulting cubic inequality, no working was shown with some simply stating the factors. Candidates were expected to use the remainder theorem or show some kind of long division, or just find one linear factor and compare coefficients to find the other quadratic/linear factors. Quite a number of candidates who ended up with a cubic inequality involving a negative coefficient of x^3 forgot to add a negative sign in front of their factors when factorising the cubic, thus then sketching the wrong graph and obtaining wrong ranges for x . A minority multiplied the given inequality by x^2 but found difficulty in sketching the graph of the resulting polynomial. Others sketched the quadratic on the LHS and the rational function on the RHS, found the points of intersection and solved the inequality using a graphical method.
- Difficulty was also shown in answering part (b), about Arithmetic and Geometric Progressions. Quite a number of candidates (incorrectly) assumed that the first term of both series was the same. Others added the sum of the first three and the sum of the first five terms instead of adding the values of the third and the

fifth terms of the Arithmetic Progression. Those who realised they had to equate the common ratio $\frac{a+7d}{a+3d} = \frac{a+3d}{a+d}$ without involving any additional unknowns generally managed to reach the correct answers.

- Question 7. Most candidates did very well in this question, in fact 64% got 5 marks or more. However, there were some common issues. In particular, it was the limits which caused problems in the first integral, rather than the substitution. Most candidates left them in terms of x and then did mistakes when converting back the integral obtained to that in terms of x . For the second integral many candidates had problems integrating by parts. A significant number of them did not manage to express the answer in the required form.
- Question 8. Most candidates answered part (a) correctly but most of them got a bit stuck in the second part. Very few candidates managed to obtain a good sketch, although not required, and so they were a bit lost. The most common mistake was using the wrong pair of lines/points to get the length of the diagonal. This compromised the rest of the solution although the vast majority managed to obtain the equation of a circle with the diameter found in part (b).
- Question 9. A good number of candidates managed to answer part (a) correctly, but others still seem to confuse permutations and combinations. Others did not read the question properly, and used the partitions method to divide the 8 posters in twelve classrooms. The majority found difficulty in answering part (b), with some misunderstanding the question completely. Most did not realise that the first digit of the number formed could not be zero. Others took only 4 digit numbers into consideration. Very few managed to match the given answer. The few candidates who added the four expected terms found it hard to simplify their answer in the given form, and added up the terms and matched it with the (worked-out) answer of the given expression. Very few answered part (b)(ii) correctly. It was also noted that in quite a number of cases, the BIDMAS rule was not being followed when evaluating the given term.
- Question 10. Candidates did not perform well in this question, with some not even attempting it (probably due to the fact that it was the last question and they ran out of time). Most candidates managed to find the correct values of R and Y_0 . Less candidates managed to find the correct value for w , and very few found the correct value for α , with most stating it was $\frac{\pi}{12}$. In part (b), quite a number of candidates did not realise that they just had to expand the given equation using trigonometric identities, substitute the values found in (a), and simply compare coefficients to find the constants A and B .

The equation the candidates were expected to solve in the last part of (b) contained a misprint. Thus, those who managed to find the correct values for the

unknowns in (a) and the first part of (b), and who also attempted to solve this equation correctly, were given full marks for this last part, even if they could not reach the correct answers. Others were also given method marks for substituting correctly and using some trigonometric identity to try and solve the resulting equation.

Paper 2

Question 1. Although there were roughly the same number of candidates attempting this question as last year (95%), the scores were significantly higher. The average mark was 12 as compared to 10 last year. The two differential equations were not difficult to solve, yet about 10% scored less than 5 marks and showed a very limited understanding of ordinary differential equations.

In part (a) a small number of candidates did not simplify the expression of the integrating factor, i.e. $e^{\ln(\sin x)} = \sin x$ and this produced many difficulties in the integration that followed. Surprisingly some had trouble in finding $\int \sin x \cos x \, dx$. Part (b) was in general very well tackled, but a few did not find the derivatives of Cx^2e^{3x} correctly.

Question 2. This question was attempted by nearly all candidates and many of them obtained high marks. Candidates knew how to use the Newton-Raphson method although a few could not differentiate correctly $\sin(x^2 + 1) - 2e^x + 3$ and others found difficulty in expanding the series $\frac{81}{(9+x^5)}$.

Question 3. Candidates worked out correctly part (a)(i) of the question but some of them were unable to show that $I_n = \frac{1}{3}x^3(\ln x)^n - \frac{n}{3}I_{n-1}$. Some candidates tried unsuccessfully to find the volume of the solid generated by the given curve without using part (a)(ii). Very few were able to find the surface area of the tyre-shaped balloon.

Question 4. Most candidates attempted this question. There were candidates who did not find the equation of the oblique asymptote and/or the nature of the stationary points. Only a few managed to work out correctly part (e).

Question 5. In general, part (a)(i) of this question was proved correctly; however only a few managed to find the sum to infinity in part (a)(ii). The vast majority of candidates did not manage to sketch the curve with polar equation $f(\theta) = 1 - 2\sin 3\theta$.

Question 6. Many of the candidates who attempted this question obtained low marks. They were unable to find the equation of the plane Π_2 and to find the values of α and β in the equation of the plane $2x + \alpha y + \beta z = -6$.

- Question 7. This was not a popular question and attempted only by 14% of the candidates. Of those trying it, 74% scored less than 5 marks, and only 2 candidates managed to obtain the full marks. In fact 14% of those who tried this question attempted only part (a) which involved a simple manipulation of a quadratic expression and obtained the 4 marks allocated to this part of the question. In part (b) most students made the error of treating the complex number w as a real number in the equation $|z| = k|z - w|$, and thus could not find the required locus. In part (c) of the question only 4 candidates managed to obtain the correct values of k , w and λ , by using the result in part (b), and hence the maximum value of $|u|$.
- Question 8. Poorly attempted by 73%. Most candidates showed a very limited understanding and practice when dealing with summations of series. 47% scored less than 5 marks while only 4% obtained full marks. The average mark was 7. The induction part in part (a) was well tackled by most candidates, but a significant number wrote down $(k + 1)$ for the $(k + 1)^{\text{th}}$ term instead of $(k + 1)^2$. Part (b) was badly answered. The summation given in this part was simply 9 times the summation given in part (a), but many substituted erroneously $3n$ for n in the expression $\frac{n}{6}(n + 1)(2n + 1)$ given in part (a). Part (c) showed clearly how weak many students are when dealing with summations. For instance, many did not seem to realize that $A_n = \sum_{r=1}^n (3r - 2)^2$ and not $A_n = (3n - 2)^2$, and similarly for B_n .
- Question 9. Attempted by only half of the candidates with an average mark of 4.8. In part (a)(i) almost all candidates did not apply the condition $|\mathbf{A}_t| \neq 0$ to find the values of t for which the system has a unique solution. Instead they tried to solve $\mathbf{A}_t \mathbf{x} = \mathbf{u}_t$ without any success to find the values of t , i.e. all $t \neq 0, t \neq 75$. In part (a)(ii) the candidates did not seem to understand the question. Nobody tried to show that the system of equations is inconsistent for $t = 0$ and $t = 75$. In part (a)(iii) many went through all the trouble of finding \mathbf{A}_t^{-1} instead of finding directly \mathbf{A}_1^{-1} .
Part (b)(i) was very well answered. Most candidates obtained the correct equation of the required plane. However, part (b)(ii) was badly tackled. Some candidates used $\mathbf{X} = \mathbf{A}_1 \mathbf{x}$, while others used $\mathbf{x} = \mathbf{A}_1^{-1} \mathbf{X}$, but in both cases they made many errors in choosing the image/object planes.
- Question 10. Probability continues to be an area of difficulty with even the simpler parts of the question inaccessible to many candidates. Only 12% scored more than 8 marks with one candidate obtaining full marks. Most candidates just quoted the values of the required probabilities without any explanation. In part (a) almost all candidates quoted the probability $1/63$, but ignored the number of ways in which the 3 dice show the same number, i.e. 6. In part (b), the probability quoted was $5/63$, but again there are 6 ways in which 2 of the dice show the same number and also there are 3 ways in which 2 of the dice can be chosen out of 3. In part (c) the

candidates did not realize that this is a conditional probability. Part (d) was very badly answered with only one success.