L-Università ta' Malta

MATSEC
Examinations Board

Examiners' Report

Advanced Level Pure Mathematics

Main Session 2021

Summary of Results

Grade	A	B	C	D	E	F	Abs	Total
Number	44	101	136	26	57	140	50	554
%	7.9	18.2	24.6	4.7	10.3	25.3	9.0	100

Grade	A - C	A - E	FAIL
Number	281	364	140
%	50.7	65.7	25.3

Comments on Candidates' Performance

Paper 1

Question 1. A significant number of candidates did well in this question. In fact, more than half of the candidates got 5 marks or more. However, many did the same mistake when integrating $\frac{1}{1+3x}$, forgetting $\frac{1}{3}$ in the answer. Moreover, many candidates, even those who integrated both sides correctly, were not able to express the answer in the required form. One can add that there were some candidates who did not recognise the equation to be a "separable variables" type differential equation, and in some cases these were candidates who did well in the other questions.

Question 2. The vast majority of candidates did very well in this question. In fact, around $\frac{1}{3}$ got more than 8 marks. In part (a) most of the candidates differentiated the given expression correctly although some of them found it a bit difficult to put everything together in order to show that the given equation holds. Some candidates thought they had to solve a second order differential equation. In part (b) most of the candidates who tackled this question did the implicit differentiation correctly but in the vast majority of cases they proceeded to find the required equation using the gradient of the tangent instead of that of the normal as was required.

Question 3. Although most candidates who tackled this question drew a correct diagram, only a few managed to tackle it properly. However, those candidates who knew the technique to solve such a problem did well and in the majority of cases got full marks. In fact 12% of candidates got more than 8 marks.

Question 4. The majority of candidates did fairly well in this question and more than half of them got 5 marks or more. In part (a) many candidates managed to get the correct expression for the composite function $f \circ g$ but failed to get the correct domain, or in most cases, failed to show how the correct domain was obtained. In part (b) many candidates did well although a significant number got mixed up in the algebraic steps leading to the expression for the inverse function. One can add that many candidates did not answer the last part of (b), namely to give the range of f which should have been easily deduced from the previous answer.

Question 5. The majority of candidates did not perform very well in this question. In fact, more than half obtained less than 5 marks. In part (a) the majority of candidates cross-multiplied the equation and then got mixed up in the steps that followed. Moreover, most of those who managed to get the correct values of a and b found it difficult to evaluate $\frac{1}{a+ib}$. This led them to wrong values of α and β. In part (b) the majority of candidates managed to obtain the correct expression but very few managed to find the smallest positive angle which gives the maximum value of the given expression. One can add that although the range for α was given in degrees, the vast majority of candidates gave the value of α in radians.

Question 6. In part (a) of this question, various methods were applied by candidates to find k and factorise the given function. Quite a number of candidates did not apply the factor theorem to find k and performed a long division in terms of k, which generally led to many errors. It was also noted that a considerable number of candidates did not know how to factorise a quadratic. In part (b), very few managed to prove the first part correctly. A lot of candidates stated, without giving any reasons, the values of $\log_4 32$ and $\log_8 4$. The majority did reasonably well in the remaining part, however there was still a considerable number of candidates who did not know how to apply the law of logarithms.

Question 7. The majority did well in part (a) of this question. However some did not apply any substitution as required, or differentiated incorrectly. Candidates also performed well in part (b). It was noted that the majority who lost marks in this part of the question were those who did not show separate working for the second integration by parts, but tried to substitute the second integral directly in the first. This resulted in wrong signs and missing multiples, thus leading to an incorrect answer.

Question 8. In part (a) of this question, most candidates managed to expand the given expression correctly. However, since very few found the range of values of x for which the expansion was valid, the majority did not substitute a correct value of x to find $\sqrt[3]{999}$. In part (b), a substantial number of candidates showed poor knowledge of the sum of a GP, with many forgetting to change the inequality sign when multiplying by a negative number. Quite a number of candidates listed and added the individual terms of the given GP to find the required number of terms.

Question 9. Candidates did not perform well at all in this question. Most of them seem to remember the transformation matrices by heart, having no idea how these are deduced from the images of $(1,0)$ and $(0,1)$. In fact, candidates used the transformation matrices to find the images of $(1,0)$ and $(0,1)$, instead of working the other way round. Poor knowledge of composite transformations was shown in part (d), with quite a number multiplying the matrices the wrong way round. In part (e), very few candidates realised that the given transformations, being rotations, were commutative.

Question 10. Candidates did reasonably well in part (a) of this question. However, some did not find the new intercepts but simply stated what kind of transformation was being applied, while others just assumed that the given function was the equation of a line. In part (b), the overall performance was good. Difficulty was generally found in part (ii), yet a good number of candidates managed to calculate or list the correct number of palindromes. Some wrongly assumed that a binary number has to start with a 1, defeating what was being asked for in part (iii).

Paper 2

Question 1. This was a very popular question attempted by 93% of the candidates with 25% of them obtaining full marks. In part (a) of the question some candidates did not simplify the integrating factor, that is $e^{3\ln(x)} = x^3$ and led to many errors in the solution. In part (b) the majority of candidates found no difficulties in solving the given second order differential equation. Although the two differential equations were straightforward, about 19% of the attempts indicated a very poor understanding of ordinary differential equations and scored no marks.

Question 2. Most candidates attempted this question and many obtained high marks for their work. In part (a) of the question there were a few candidates who although familiar with the Newton-Raphson method were unable to differentiate $f(x) = x - e^{-x^2}$. In part (b) most candidates did not find any difficulties in evaluating the given integral using Simpson's Rule.

Question 3. Part (a)(i) of the question was well answered by the candidates using different methods. However, although a hint was given, only few candidates were able to integrate $\cot^n x$ in (ii) to obtain the required result. Many of the candidates used correctly the result of part (a)(ii) to solve (a)(iii). Part (b) was correctly answered by most of the candidates.

Question 4. This was a well attempted question. However, although 73% of the candidates attempted this question, only 3% obtained full marks. In part (a), many candidates obtained the correct partial fractions in (i), but adopted a very long method to find the required constants when they could have easily written them down by using the *the cover up* method. In (ii), most of those candidates who obtained the correct partial fractions found no difficulties in finding S_n and S_∞. In (iii), only a few number of candidates managed to show that the given series is equal to $\sum_{r=1}^{10} \frac{3r+4}{r^3+3r^2+2r} - \frac{7}{6}$. There was a large number of candidates who wrote down the correct answer without giving any working. Part (b) of the question was poorly attempted. Most of the candidates failed to write down the coefficient of x^n in the Maclaurin's series which is simply $\frac{f^{(n)}(0)}{n}$.

Question 5. This question was attempted by 55% of the candidates, but only 2% obtained full marks. In part (a) of the question, the first summation was very well tackled and most of the candidates obtained the correct expression for the sum. A frequent error was that $\sum_{r=1}^{n} 4 = 4$. In the second summation many candidates found no difficulties in showing that its sum is given by $\sum_{r=2}^{10}(3r-2)^2-1$. Quite a few candidates just wrote down the answer without showing any working. Part (b) of the question proved to be more difficult than expected. In (i) most candidates used the correct substitutions $x = r\cos\theta$, $y = r\sin\theta$, but a few concluded incorrectly that $r^2-2r\cos\theta-8r\sin\theta = 0$ is the polar equation of the circle and although they sketched the circle, they did not observe that $r \neq 0$ for all values of θ and so it was possible to divide by r. (ii) was very poorly answered. A common error made in finding the required area was while integrating the above expression equating $\int \sin^2\theta \, d\theta$ to $\frac{\sin^3\theta}{3}$.

Question 6. Some of the candidates who answered this question did not know the formula for finding the volume of a parallelepiped. Many candidates managed to find the equation of the plane but were unable to find the point of intersection of the given line and the plane.

Question 7. This was a very unpopular question. It was attempted by only 90 out of 500 candidates and they all scored less than 9 marks. In part (a)(i), most of these candidates produced only fragmentary results. They found difficulties in deducing that $f(z) - g(z) =$ constant if and only if $a = b$, and that the equation $f(z) \cdot g(z) = 0$ has no real solution implies that $a^2 - 52 < 0$ and $b^2 - 8 < 0$. In (ii) very few candidates noted that the given equation corresponds to the equation $f(z) \cdot g(z) = 0$ with $a = b = 2$. In part (b) nobody defined $\sin x$ and $\cos x$ in terms of e^{ix} correctly and made many basic errors in trying to find the value of k.

Question 8. This question was only attempted by 40% of candidates and nobody managed to obtain the whole 15 marks. Part (a) was the only part of the question that was tackled correctly. The way summations are handled is quite good with the method of mathematical induction reduced to a very systematic process by most candidates. In part (b) almost all candidates tried to use the method of mathematical induction to prove the inequality treating x as an integer. Only 3 candidates simplified the given inequality to $1-x \leq 0$ which is true since $x \geq 1$. In part (c) most of the candidates treated the given product as a sum. They added the $(k+1)^{\text{th}}$ term, that is $\frac{2k+1}{2(k+1)}$, instead of multiplying and did not use part (b).

Question 9. In this question, to prove that the equations are consistent, there were candidates who just subtracted the sum of 4 times the equation containing λ_2 and the equation containing λ_3 from the equation containing λ_1 without doing row reduction. In working part (b) of the question, candidates committed mistakes when performing row operations on the matrices.

Question 10. Many candidates attempted this question. Some of the candidates were unable to work out the equation for the oblique asymptote but most candidates found correctly were the curve cuts the coordinate axes. The curve of the function $f(x)$ was well sketched by many candidates but only a few sketched $\frac{1}{f(x)}$ correctly.

Chairperson
Examination Panel 2021