L-Università ta' Malta

MATSEC
Examinations Board

# Examiners' Report

## AM Pure Mathematics

## First Session 2022

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## A. STATISTICAL INFORMATION

The total number of candidates who registered to sit for AM Pure Mathematics was 514.

The following table shows the distribution of grades for the First 2022 session of the examination

| GRADE | A | B | C | D | E | F | ABS | TOTAL |
|---|---|---|---|---|---|---|---|---|
| NUMBER | 31 | 93 | 105 | 27 | 52 | 149 | 57 | 514 |
| % OF TOTAL | 6.0 | 18.1 | 20.4 | 5.3 | 10.1 | 39.0 | 11.1 | 110 |

| Grade | A - C | A - E | FAIL |
|---|---|---|---|
| Number | 229 | 308 | 149 |
| % | 44.5 | 59.9 | 29.0 |

## B. COMMENTS ON PAPER I AND PAPER II

### Paper I

*Question 1*

Almost half of the candidates had difficulties in rearranging the given equation to separate the variables $x$ and $y$ and thus continue with the integration. Half of those whore arranged well the given expression managed to get the correct solution, while the other half got stuck in the integration. A significant number of candidates had difficulties giving their answer in the required form. A very common mistake was that of writing the integral of ln2 as ln2$x$ instead of (ln2)$x$, which then by mistake was considered as ln(2$x$). There were some candidates who integrated the constant $k$ as $\frac{1}{2}k^2$.

*Question 2*

The vast majority of candidates did well in part (a), where they had to differentiate implicitly the given equation. However, some got lost in the process of finding the coordinates of the stationary points. Almost half of the candidates either did not attempt part (b), or found difficulty to get the second derivative. Around 40% of the other candidates got stuck while finding the second derivative, mainly because they did not use the fact that at the stationary points, $y' = 0$. Many candidates did well in part (c), even those who got stuck in part (b) since they could use the given expression in part (b) to solve part (c).

*Question 3*

The vast majority of candidates did very well in this question. In fact, 56% got more than 8 marks, while 65% got more than 5 marks. One can note that there were some candidates who managed to use the scalar product correctly to show that the lines were perpendicular in part (b) but then failed to use it properly in part (c) to find the angle required and vice-versa, where basically it is the same application of the scalar product.

### Question 4

Candidates showed a significant lack of algebraic skills in this question which limited their ability to answer it properly. In part (a) the vast majority of candidates failed to see that to get $f(x)$ from $f(x+1)$, one simply had to substitute $x$ by $x - 1$ in the given expression. In part (b) and part (c) about half of the candidates managed to get the right expression for the requested functions and even for the domain of $g \circ f$. However, one should note that very few candidates gave the correct domain of $f^{-1}$ in part (b).

### Question 5

In part (a), many candidates managed to prove the first identity but failed to prove the second one. However, there was a good number of candidates who managed to prove both but failed to evaluate $\sin(A + B)$, or in some cases did not even attempt it. In part (b), the performance was much worse with only around 10 out of 514 candidates solving the trigonometric equation without first cancelling the common term $\sin 2\theta$, and thus getting less than half the required solutions.

### Question 6

In part (a) of this question, the most common mistake was to take the series as a geometric series instead of an arithmetic series. Many candidates arrived at forming the correct equation in powers of $x$, but failed to treat it as a quadratic to be able to solve it. In part (b), many candidates used $b^2 - 4ac \geq 0$ for two roots instead of $> 0$. Although many candidates found the values of $k$, many of them did not sketch the graph and so did not find the required range of values of $k$.

### Question 7

In part (a) of the question the majority of candidates proved the given identity correctly. However, some stopped short of replacing $\tan \frac{\pi}{4}$ by 1 or simplified badly the resulting fraction. In part (b) many candidates made the required substitution but quite a few just changed $d\theta$ to $d\phi$. The majority of candidates did not realise that the required integral $I = \frac{\pi}{4} \ln 2 - I$ so that they went round in circles. There were different ways to tackle part (c) of this question. Many candidates did not change $dx$ to $\sec^2\theta \, d\theta$ during the substitution and there were others who did not change the limits.

### Question 8

In part (a) of this question, many candidates obtained the inequality $x > 1/3$ by just cross-multiplication, without considering the case when $x+2$ is negative. The candidates who got the question correct either drew a sketch of $y = 3x^2+5x-2$ and found the correct range or else considered also when $x+2$ is negative reversing the inequality sign. In part (b), the majority of candidates could prove easily that $2x-1$ is a factor by finding that $f(1/2) = 0$ or by performing a division to show that there is no remainder. However, quite a few candidates did not factorize the quotient to $(x+2)^2$. This made it impossible to find the partial fractions. Quite a few candidates used $Ax+B$ in the numerator, when $A$ was enough, and then obtained $A = 0$.

*Question 9*

Very few candidates solved part (a) of this question completely. Some got the correct total number of possibilities; however, the rest of the reasoning was not correct. In part (b)(i) most candidates proved the required identity, but others just wrote down the answer or tried out some numbers for *a* and *b*. In part (b)(ii) many candidates could find the inverse of the matrix **P** correctly but some did not divide by the determinant of **P**. In part (b)(iii) very few candidates arrived at the correct answer for matrix **A**. A large number assumed wrongly that matrix multiplication is commutative and went out to deduce $\mathbf{A}=\mathbf{D}^3$.

*Question 10*

Although the majority of candidates expanded the expression $(x - 2)^3$ correctly in part (a) of this question either by using Pascal triangle or by long multiplication, many failed to deduce the graph of $y = (x - 2)^3 + 8$. Many candidates showed a maximum and a minimum instead of a point of inflexion. Also, the reason for the function $x^3 - 6x^2 + 11x$ to have only one real root was given by some that it should be similar to the one they drew instead of seeing where their graph cuts the line $y = x$. In part (b) most candidates started the question correctly with the correct algebraic expressions for the lengths of *PA* and *PB*. However, when they came to squaring in order to remove the terms with a square root, they left out the term resulting with another square root.

## Paper II

*Question 1*

This was a very popular question attempted by 95% of the candidates with 18% of them obtaining full marks. In part (a) of the question some candidates made the error of taking the integrating factor to be $e^{\int 2\tan x \, dx}$ instead of $e^{-\int 2\tan x \, dx}$. In part (b) the majority of candidates found no difficulties in solving the given second order differential equation.

*Question 2*

This question was answered by nearly all of the candidates. The candidates seemed to be familiar with the Newton-Raphson method and with Simpson's Rule. However, there were some candidates who were unable to differentiate correctly the function $f(x) = 2\ln(1+\sin^2 x) - \cos x$. Many candidates obtained high marks for this question.

*Question 3*

Many candidates did not choose to answer this question. Evaluating the integral of $(1 - x^2)^{\frac{3}{2}}$ was beyond the ability of some candidates. In part (c) of the question, finding the derivative of $\frac{x^4}{4} + \frac{1}{8x^2}$ and the area of the surface of revolution of part of the given curve caused many difficulties to the candidates and were not answered correctly by a good number of them.

*Question 4*

Only 5 candidates got full marks for this question which was attempted by 54% of the candidates. This question was very poorly answered and in fact, 17% got no marks at all. In part (a) most candidates used the quotient rule for differentiation to find the first four derivatives of the given function. It would have been easier to use the function of a function rule for differentiation. In part (a)(ii), quite a number of candidates substituted for $x$ the given value of $\frac{8}{9}$ in their obtained series. In part (b)(i) most of the candidates did not use the basic properties of the ln function. Hence, they did not realize that this problem is about summation of a series using the method of differences.

*Question 5*

This question was attempted by about 50% of the candidates out of which only 2% got full marks. In part (a)(i), some candidates did not simplify well enough the required equation. In fact, they ended up with $r = 4\sin\theta\cos\theta$, which should have been further reduced to $r = 2\sin 2\theta$. Problems then arose when sketching the curve. In part (a)(ii) some candidates did not notice that the range to sketch the curve was for $0 \leq \theta \leq \pi$. Some candidates did not know how to tackle negative values of $r$. In part (b)(i), a common mistake made by candidates was that they $\sum_{r=1}^{n} 27 = 27$ and not $27n$. In part (b)(ii), most of the candidates did not notice that there was the word *Hence*, and that they needed to use the result they obtained in part (b)(i) and not use a calculator.

*Question 6*

This was a question on the use of vectors. While trying to solve part (a), some candidates were unable to find a vector perpendicular to the plane and so could not work out correctly the equation of the plane that passes through point $A$ and containing the given line. Part (b), which did not depend on part (a), was answered correctly by most of the candidates. The other parts of the question depended on the answers of part (a) and/or (b) to be answered completely.

*Question 7*

This question was very poorly attempted. In fact, only 33% of the candidates chose this question, and only two candidates got all of it correct. In part (a), some candidates forgot to square the constant 2 when working out the modulus of the given equation. There were other candidates who did not divide by 3 when they were trying to find the centre and radius of the required circle. In part (b), many candidates did not realize that when taking the cube root of the equation $(2z+1)^3 = (z+2i)^3$, they should get $(2z+1) = \sqrt[3]{1}\,(z+2i)$. Then they needed to find the three cube roots, two of which are complex, of 1.

*Question 8*

About 60% of the candidates attempted this question and 11% of these candidates obtained full marks. It was a standard question and the candidates had to follow the process of mathematical induction. There were some candidates who did not show that the given result holds for the basic case. It seemed to be quite difficult for some candidates to simplify two algebraic terms by taking what is common. In fact, they expanded the terms and got confused in simplifying.

*Question 9*

A good number of candidates did not attempt this question. There were candidates who made mistakes when working out the row operations and thus obtaining the wrong answers. Many of those candidates who did answer this question were unable to find the range of values of $k$ for which the planes meet at exactly one point.

*Question 10*

This was a question on curve sketching. The majority of candidates found the vertical asymptotes and the points where the curve cuts the axes correctly. However, there were quite a few candidates who did not obtain correctly the horizontal asymptote. Others did not manage to show that there were no stationary points. The sketching of the curves $f(x)$ and $\frac{1}{f(x)}$ on the same diagram was not done clearly by many candidates.

Chairperson
Examination Panel 2022