L-Università ta' Malta

MATSEC Examinations Board

IM 02 SYLLABUS

Applied Mathematics

2028

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Introduction

Applied Mathematics connects mathematical theory to practical uses in science, engineering, and everyday problem-solving. This syllabus serves as an introduction to mathematics as applied to classical mechanics. It assumes that candidates already have a solid grounding in mathematics at the Secondary Education Certificate level and builds on that foundation to develop a deeper understanding of methods and techniques that can be applied in real-world contexts. The course focuses on strengthening analytical thinking, logical reasoning, and quantitative skills, enabling learners to model, interpret, and solve problems.

The syllabus is structured into seven Learning Outcomes, each supported by Assessment Criteria to help teachers, candidates, and examiners gauge the appropriate depth and breadth of study. While teachers may extend lessons beyond these outcomes, examinations will focus on the content as outlined. The course aims to prepare candidates for further study, equipping them with both the theoretical insight and practical competence needed to apply mathematics effectively.

List of Subject Foci

1. Vectors
2. Statics
3. Kinematics in 1D
4. Dynamics of a Particle
5. Mechanical Work, Energy and Power
6. Momentum
7. Motion of a Particle in 2D

List of Learning Outcomes

At the end of the programme,

LO 1. I can represent vectors in two dimensions and perform vector operations.

LO 2. I can analyse systems of coplanar forces acting on particles and rigid bodies to determine conditions for equilibrium.

LO 3. I can use graphical and algebraic methods to model and solve problems involving rectilinear motion under constant acceleration.

LO 4. I can apply Newton's Laws of Motion to solve problems involving motion of particles of constant mass and to describe the forces.

LO 5. I can apply the principles related to work-energy and the conservation of energy.

LO 6. I can apply the principle of conservation of linear momentum, and the law of restitution for direct impact.

LO 7. I can solve problems related to the motion of a particle in a horizontal circle and to projectiles.

Preamble

1. Vector cross product is not required.
2. Knowledge of differentiation and integration of functions is not expected.
3. SI units are to be used unless otherwise specified.
4. Knowledge of angle measurement in radians is required.
5. Competence in the conversion of units is expected - for example g to kg, km/h to m/s, deg to rad, rev/min to rad/s.
6. The use of a calculator is allowed.
7. Unless otherwise stated, the following should be assumed:
    a. particles are to be treated as point masses;
    b. all bodies are rigid;
    c. rods are 1D and light;
    d. laminas are 2D;
    e. strings are light and inextensible;
    f. pulleys are light and smooth;
    g. hinges and pegs are smooth;
    h. surfaces are smooth;
    i. uniform motion refers to motion at constant velocity;
    j. air resistance is ignored.
8. The value of the acceleration due to gravity, g, is to be taken as the constant 10 m/s^2.
9. Knowledge of the Mathematics Syllabus for Level 3/3* is assumed.
10. Use of the following trigonometric identities and relations is required:
    a. The three reciprocal ratios: $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$; $\cot\theta = \frac{1}{\tan\theta}$
    b. $\sin\theta = \cos(90-\theta)$ and $\sin(90-\theta) = \cos\theta$
    c. $\sin(180-\theta) = \sin\theta$ and $\sin(90+\theta) = \cos\theta$
    d. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
    e. The sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$
    f. The Pythagorean identities: $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$
    g. The double angle identity for $\sin 2\theta = 2\sin\theta\cos\theta$
    h. In the range 0 to 180°, the equation $\sin\theta = k$ has two solutions (for k positive) while the equation $\cos\theta = k$ has one solution

Learning Outcomes and Assessment Criteria

Subject Focus:	**Vectors**
Learning Outcome 1:	**I can represent vectors in two dimensions and perform vector operations.**

Topic	Sub-Topic	Assessment Criteria
1.1 Vectors	1.1.1 Operations with Vectors	1. Distinguish between scalars and vectors. 2. Find the magnitude and direction of a vector. 3. Perform vector addition, subtraction and scalar multiplication. 4. Represent points using position vectors. 5. Use the unit vectors **i**, **j**. 6. Recognise like parallel vectors and unlike parallel vectors. 7. Calculate the dot product of two vectors.
	1.1.2 Resolution and Resultant of Vectors	8. Resolve a vector into two perpendicular components. 9. Calculate the magnitude and direction of the resultant of two or more coplanar vectors.
	1.1.3 Vector Equation of a Straight Line	10. Write the vector equation of a straight line given a point on the line and a direction vector. 11. Identify the initial point and the direction vector in the vector equation of a line for a particle moving with constant velocity. 12. Convert a vector equation of a line to a 2D Cartesian equation and vice versa. 13. Determine whether two lines are parallel or intersecting. 14. Find the point of intersection of two lines. 15. Solve problems involving uniform motion using vectors.

Subject Focus:	**Statics**
Learning Outcome 2:	**I can analyse systems of coplanar forces acting on particles and rigid bodies to determine conditions for equilibrium.**

Topic	Sub-Topic	Assessment Criteria
2.1 Co-Planar Forces	2.1.1 Forces	1. Apply the concept of force as a vector. 2. Apply the parallelogram and triangle rules for vector addition of forces. 3. Determine the vector or/and Cartesian equation/s of the line of action of a force. 4. Determine the resultant of a system of coplanar forces acting at a point.
	2.1.2 Static Force Analysis	5. Solve problems involving concurrent forces in equilibrium using one of the following methods: a) resolve into components; b) use the triangle of forces; c) use Lami's Theorem. 6. Sketch a labelled force diagram that includes the external and internal forces acting on a body. 7. Solve problems involving compression/tension forces in light rods and tension forces within inextensible strings.
	2.1.3 Friction	8. Apply the concept of limiting equilibrium, the angle of friction and the coefficient of friction. 9. Apply the inequality $F \leq \mu R$ in solving equilibrium problems. 10. Solve problems involving normal reaction, frictional forces, coefficient of friction and other external forces.

Topic	Sub-Topic	Assessment Criteria
	2.1.4 Moments of Forces	11. Calculate the moment of a force, or a system of coplanar forces, about an axis perpendicular to the plane containing the forces. 12. Solve problems involving parallel forces and couples. 13. Reduce a system of forces to an equivalent single force, a couple, or a single force and a couple.
	2.1.5 Hooke's Law	14. Apply Hooke's Law using the modulus of elasticity, in the form $T = \frac{\lambda x}{l}$. 15. Solve problems involving the extension or compression of springs and strings.
	2.1.6 Framework Analysis	16. Sketch a labelled diagram of a framework made of light, smoothly jointed rods. 17. Solve static equilibrium problems to determine unknown forces in a framework. 18. Determine whether rods in a framework are in tension or compression.
	2.1.7 Centre of Mass of Particles and Uniform Bodies	19. Calculate the centre of mass of a group of particles arranged in two dimensions. 20. Use symmetry and basic geometry to find the centre of mass of composite bodies. *The addition and subtraction of a region is expected.* 21. Solve problems involving a rigid body suspended from a fixed point, including finding an angle with the vertical. *The solid body must have a plane of symmetry to reduce to a 2D problem.* *Problems on sliding and toppling are not expected.*

Subject Focus:	**Kinematics in 1D**
Learning Outcome 3:	**I can use graphical and algebraic methods to model and solve problems involving rectilinear motion under constant acceleration.**

Topic	Sub-Topic	Assessment Criteria
3.1 Uniformly accelerated motion	3.1.1 Graphical and Algebraic Representation of Rectilinear Motion	1. Apply kinematic equations for motion under constant acceleration. 2. Sketch displacement–time graphs for problems limited to constant velocity. 3. Sketch velocity–time graphs for problems limited to constant acceleration. 4. Interpret displacement-time graph. *To understand that a horizontal line implies the body is at rest and that the gradient of an inclined straight line implies motion at a constant velocity.* 5. Interpret velocity-time graph. *To include: the calculation of the displacement/distance travelled as the area under the graph; and of the acceleration as a gradient/slope of the graph.* 6. Analyse motion under gravity in 1D in a non-resistive medium.

Subject Focus:	**Dynamics of a Particle**
Learning Outcome 4:	**I can apply Newton's laws of Motion to solve problems involving motion of particles of constant mass and to describe the forces.**

Topic	Sub-Topic	Assessment Criteria
4.1 Newton's Laws of Motion		1. Apply Newton's laws of motion. 2. Recognise that a force on a body of constant mass causes an acceleration, that is a change in velocity. 3. Distinguish between mass and weight. 4. Solve problems related to motion in a straight line for bodies of constant mass moving under the action of constant forces, using $F = ma$. 5. Sketch free-body diagrams of forces, also showing velocity and acceleration. *Applications may include:* *a) a body moving along a horizontal or an inclined plane;* *b) a body travelling in a lift;* *c) a train engine pulling a carriage along a straight horizontal or inclined track;* *d) a crane lifting a weight;* *e) a ball falling under gravity;* *f) a particle moving on a rough or smooth inclined plane under the action of gravity, friction, normal reaction, and pulling/pushing forces, where all the forces and motion are in the same vertical plane as the line of greatest slope;* *g) the vertical motion of two particles connected by a light inextensible string passing over a fixed smooth peg or light pulley;* *h) the reaction force between two bodies becomes zero, when contact is lost.*

Subject Focus:	**Mechanical Work, Energy, and Power**
Learning Outcome 5:	**I can apply the principles related to work-energy and the conservation of energy.**

Topic	Sub-Topic	Assessment Criteria
5.1 Energy, Work, and Power	5.1.1 Work	1. Calculate work done using the formulae $W = Fd\cos\theta$ or $W = \boldsymbol{F}.\boldsymbol{s}$ 2. Calculate work done by/against specific forces (e.g., gravity, friction, tension, applied force).
	5.1.2 Mechanical Energy	3. Recall formulae for Kinetic Energy $(KE) = \frac{1}{2}mv^2$, Gravitational Potential Energy $(GPE) = mgh$ and Elastic Potential Energy $(EPE) = \frac{\lambda x^2}{2l}$ 4. Calculate the energy stored in an elastic string or spring. 5. Apply the Work-Energy principle, that is, the total work done is equal to change in mechanical energy.
	5.1.3 The Principle of Conservation of Mechanical Energy	6. Apply the principle that KE + PE remains constant during frictionless motion (where PE = GPE + EPE is the Potential Energy). 7. Use conservation of mechanical energy to solve problems involving changes in speed, vertical position, and compression/extension. 8. Applications may include vertical circular motion of a point mass under gravity. 9. Problems may include determining the conditions under which the string becomes slack while moving in a straight line. 10. Determine the extreme values of speed (maximum) or height (minimum/maximum) of a particle in motion, for example a particle attached to an elastic spring/string.
	5.1.4 Power	11. Recognise power as the rate of doing work. 12. Use the formula $P = \frac{W}{t}$ to find the average rate of work done. 13. Recall the formula for power as the product of force and velocity acting in the same direction, $P = Fv$.

Subject Focus:	**Momentum**
Learning Outcome 6:	**I can apply the principle of conservation of linear momentum, and the law of restitution for direct impact.**

Topic	**Sub-Topic**	**Assessment Criteria**
6.1 Momentum and Impulse	6.1.1 Momentum	1. Calculate linear momentum as the product of the mass and velocity of the body. *Note: This is expected in scalar or vector form.*
	6.1.2 Impulse	2. Calculate the impulse of a constant force in a given time interval, using the formula $J = Ft$. 3. Apply the principle that impulse is equal to change in momentum.
	6.1.3 Conservation of Momentum	4. Apply the Principle of Conservation of Linear Momentum to a closed system of particles. 5. Apply conservation of momentum to solve problems involving collisions and recoil/explosions in one dimension. *Resolving is not required.* 6. Formulate momentum equations before and after an impact.
	6.1.4 Direct Impacts and Newton's Law of Restitution	7. Calculate the coefficient of restitution (e). 8. Recognise that perfectly elastic collisions $(e = 1)$ incur no loss in kinetic energy, while perfectly inelastic collisions $(e = 0)$ incur maximum loss of kinetic energy. *Examples include:* *a) direct impact between two particles;* *b) direct impact between a particle and a fixed smooth wall;* *c) successive direct impacts between three particles and/or a wall.* 9. Solve problems involving direct impacts and the coefficient of restitution. *Note: Oblique impacts are excluded.*

Subject Focus:	**Motion of a Particle in 2D**
Learning Outcome 7:	**I can solve problems related to the motion of a particle in a horizontal circle and to projectiles.**

Topic	Sub-Topic	Assessment Criteria
7.1 Uniform motion in a horizontal circle	7.1.1 Angular Speed and Centripetal Acceleration	1. Recall the formula for angular speed $\omega = \frac{\theta}{t}$. *Angular speed may be measured in both radian per second or as revolution per unit time.* 2. Recognise the relation between linear speed and angular speed, $\omega = \frac{v}{r}$. 3. Recall formulae for centripetal acceleration ($a = \frac{v^2}{r} = r\omega^2$). 4. Apply the knowledge that an object moving at constant speed in a circle is accelerating. 5. Recognise that the direction of centripetal acceleration is inwards and towards the centre of motion.
	7.1.2 Centripetal Force	6. Recognise that centripetal force is the resultant force acting towards the centre of the circle, causing the centripetal acceleration ($F = ma = m\frac{v^2}{r} = mr\omega^2$). 7. Sketch clear force diagrams for objects undergoing circular motion, indicating the direction of the resultant centripetal force. *Note that the centripetal force is not a special type of force but a role played by existing forces (e.g., tension, friction, and normal reaction).* *Applications may include:* *a) conical pendulums;* *b) aircraft flying in a horizontal circle;* *c) motion on a banked track.* *Problems are restricted to constant angular speed, and forces of constant magnitude, but resolution of forces is expected.*

Topic	Sub-Topic	Assessment Criteria
7.2 Projectiles		8. Recognise that the motion of a projectile consists of a combination of independent horizontal and vertical motions (ignoring air resistance). *The use of vector notation is expected.* 9. Recall that horizontal velocity is constant and vertical acceleration under gravity (g) is constant. 10. Solve problems involving velocity, time taken, and angle of projection for a projectile at any given time or position. 11. Solve problems involving projectiles launched from a position that has a higher, lower or same height as the landing position. *Problems on inclined planes and/or with air resistance are excluded.* 12. For a projectile starting and ending on the same level, derive the equations of: *a) the trajectory of the projectile;* *b) the time of flight;* *c) the horizontal range;* *d) the maximum horizontal range;* *e) the maximum height.*

Scheme of Assessment

This subject is assessed by means of **ONE** component: written examination paper.

The examination consists of one paper of three hours. This paper will contain ten questions each carrying between eight to twelve marks. The total number of marks is 100. Candidates will be required to answer all questions.