The impact of asynchronous communication on the theory of contracts and session types

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Department of Computer Science
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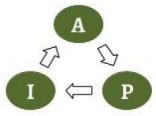
INRIA research team FOCUS

Based on joint work with:
Mario Bravetti, Marco Carbone, Nobuko Yoshida, Julien Lange

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 778233



BEHAPI
Behavioural Application Program Interfaces

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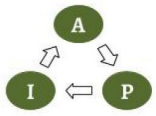

BEHAPI
Behavioural Application Program Interfaces

Structure of the course

- Session types: subtyping
- Behavioural contracts: CFSMs, BPA
- The synchronous case
  - Session subtyping ≈ Contract refinement
- Asynchronous communication
  - Asynchronous session subtyping
  - Async. subtyping ≈ Async. refinement
  - Positive and Negative implications

Session types [ESOP98]

Language Primitives and Type Discipline for Structured Communication-Based Programming

KOHEI HONDA*, VASCO T. VASCONCELOS†, AND MAKOTO KUBO‡

- A type discipline for communication-based **concurrent** programming

$$S ::= \mathbf{nat} \mid \mathbf{bool} \mid \langle \alpha, \overline{\alpha} \rangle \mid s \mid \mu s.S$$

$$\alpha ::= \downarrow[\tilde{S}];\alpha \mid \downarrow[\alpha];\beta \mid \&\{l_1 : \alpha_1, \ldots, l_n : \alpha_n\} \mid \mathbf{1} \mid \bot$$
$$\mid \uparrow[\tilde{S}];\alpha \mid \uparrow[\alpha];\beta \mid \oplus\{l_1 : \alpha_1, \ldots, l_n : \alpha_n\} \mid t \mid \mu t.\alpha$$

(Simple) session types

- A **minimal** syntax for session types:

$$T \quad ::= \quad \oplus\{l_i : T_i\}_{i \in I} \quad | \quad \&\{l_i : T_i\}_{i \in I} \quad | \quad \mu \mathbf{t}.T \quad | \quad \mathbf{t} \quad | \quad \mathbf{end}$$

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Selection among **outputs**

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Selection among **outputs**

Branching among **inputs**

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Selection among **outputs**

Recursion

Branching among **inputs**

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Selection among **outputs**

Recursion

Termination

Branching among **inputs**

(Simple) example

- A session type for a service:

$$S_{service} = \mu \mathbf{t}.\&\{\text{add\_to\_cart} : \mathbf{t}\ ,\ \text{pay} : \mathbf{end}\}$$

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- A session type for a client:

$$T_{client} = \oplus\{\text{add\_to\_cart} : \oplus\{\text{pay} : \mathbf{end}\}\}$$

(Simple) example

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- A session type for a client:

$$T_{client} = \oplus\{\text{add\_to\_cart} : \oplus\{\text{pay} : \mathbf{end}\}\}$$

Are the client protocol and the service protocol compatible ?

Compatibility between dual types

- A protocol is always compatible with its dual (i.e. the **specular** protocol)

$$\overline{\oplus\{l_i : T_i\}_{i\in I}} = \&\{l_i : \overline{T}_i\}_{i\in I}$$
$$\overline{\&\{l_i : T_i\}_{i\in I}} = \oplus\{l_i : \overline{T}_i\}_{i\in I}$$
$$\overline{\mu \mathbf{t}.T} = \mu \mathbf{t}.\overline{T},\ \overline{\mathbf{t}} = \mathbf{t},\ \overline{\mathbf{end}} = \mathbf{end}$$

$$S = \mu \mathbf{t}.\ \&\{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}$$

$$\overline{S} = \mu \mathbf{t}.\ \oplus\{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}$$

Liskov substitution principle [ToPLaS94]

A Behavioral Notion of Subtyping

BARBARA H. LISKOV
MIT Laboratory for Computer Science
and
JEANNETTE M. WING
Carnegie Mellon University

Subtype Requirement: Let $\phi(x)$ be a property provable about objects x of type T. Then $\phi(y)$ should be true for objects y of type S where S is a subtype of T.

- All protocols with a session type which is **subtype** of the dual are compatible

Session subtyping [ActaInf05]

Simon Gay · Malcolm Hole

Subtyping for session types in the pi calculus

Definition (Synchronous Subtyping, $\leq$). *$\mathcal{R}$ is a synchronous subtyping relation whenever $(T, S) \in \mathcal{R}$ implies that:*

1. *if $T = \mathbf{end}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \mathbf{end}$;*
2. *if $T = \oplus\{l_i : T_i\}_{i \in I}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \oplus\{l_j : S_j\}_{j \in J}$, $I \subseteq J$ and $\forall i \in I. (T_i, S_i) \in \mathcal{R}$;*
3. *if $T = \&\{l_i : T_i\}_{i \in I}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \&\{l_j : S_j\}_{j \in J}$, $J \subseteq I$ and $\forall j \in J. (T_j, S_j) \in \mathcal{R}$;*
4. *if $T = \mu \mathbf{t}.T'$ then $(T'\{T/\mathbf{t}\}, S) \in \mathcal{R}$.*

T is a synchronous subtype of S, written $T \leq S$, if there is a synchronous subtyping relation $\mathcal{R}$ such that $(T, S) \in \mathcal{R}$.

Output Covariance and Input Contravariance

- Output Covariance
  - subtype may have a **subset of outputs**
  - example:

$$\oplus\{l_1 : T_1\} \leq \oplus\{l_1 : T_1, l_2 : T_2\}$$

- Input Contravariance
  - subtype may have a **superset of inputs**
  - example:

$$\&\{l_1 : T_1, l_2 : T_2\} \leq \&\{l_1 : T_1\}$$

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Our simple example

$$\overline{S} = \mu \mathbf{t}. \oplus \{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}$$
$$T = \oplus\{\text{add\_to\_cart} : \oplus\{\text{pay} : \mathbf{end}\}\}$$

? $T \leq \overline{S}$?

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? $T \leq \overline{S}$?

- A synchronous subtyping relation:

$$\{ \ (\oplus\{\text{add\_to\_cart} : \oplus\{\text{pay} : \mathbf{end}\}\} \ , \ \mu \mathbf{t}. \ \oplus \{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}),$$
$$(\oplus\{\text{pay} : \mathbf{end}\} \ , \ \mu \mathbf{t}. \ \oplus \{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}),$$
$$(\mathbf{end} \ , \ \mathbf{end}) \ \}$$

Our simple example

$$\overline{S} = \mu \mathbf{t}. \oplus \{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}$$
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- A synchronous subtyping relation:

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$$(\oplus\{\text{pay} : \mathbf{end}\} \ , \ \mu \mathbf{t}. \ \oplus \{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}),$$
$$(\mathbf{end} \ , \ \mathbf{end}) \ \}$$

  - unfold of the supertype:

$$\oplus\{\text{add\_to\_cart} : (\mu \mathbf{t}. \ \oplus \{\text{add\_to\_cart} : \mathbf{t}, \text{pay} : \mathbf{end}\}), \text{pay} : \mathbf{end}\}$$

The subtyping simulation game

$$\frac{T \leqslant U \in \Sigma}{\Sigma \vdash T \leqslant U}\ \text{AS-Assump} \qquad \frac{}{\Sigma \vdash \mathsf{end} \leqslant \mathsf{end}}\ \text{AS-End}$$

$$\frac{\Sigma, \mu X.T \leqslant U \vdash T\{\mu X.T/X\} \leqslant U}{\Sigma \vdash \mu X.T \leqslant U}\ \text{AS-RecL}$$

$$\frac{\Sigma, T \leqslant \mu X.U \vdash T \leqslant U\{\mu X.U/X\}}{\Sigma \vdash T \leqslant \mu X.U}\ \text{AS-RecR}$$

$$\frac{\Sigma \vdash \widetilde{T} \leqslant \widetilde{U} \quad \Sigma \vdash V \leqslant W}{\Sigma \vdash ?[\widetilde{T}].V \leqslant ?[\widetilde{U}].W}\ \text{AS-InS} \qquad \frac{\Sigma \vdash \widetilde{U} \leqslant \widetilde{T} \quad \Sigma \vdash V \leqslant W}{\Sigma \vdash ![\widetilde{T}].V \leqslant ![\widetilde{U}].W}\ \text{AS-OutS}$$

$$\frac{m \leqslant n \quad \forall i \in \{1, \ldots, m\}.(\Sigma \vdash S_i \leqslant T_i)}{\Sigma \vdash \&\langle l_i : S_i \rangle_{1 \leqslant i \leqslant m} \leqslant \&\langle l_i : T_i \rangle_{1 \leqslant i \leqslant n}}\ \text{AS-Branch}$$

$$\frac{m \leqslant n \quad \forall i \in \{1, \ldots, m\}.(\Sigma \vdash S_i \leqslant T_i)}{\Sigma \vdash \oplus\langle l_i : S_i \rangle_{1 \leqslant i \leqslant n} \leqslant \oplus\langle l_i : T_i \rangle_{1 \leqslant i \leqslant m}}\ \text{AS-Choice}$$

Fig. 11 Algorithmic subtyping rules

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Behavioural contracts

- An **operational** model for concurrent communicating processes

Behavioural contracts

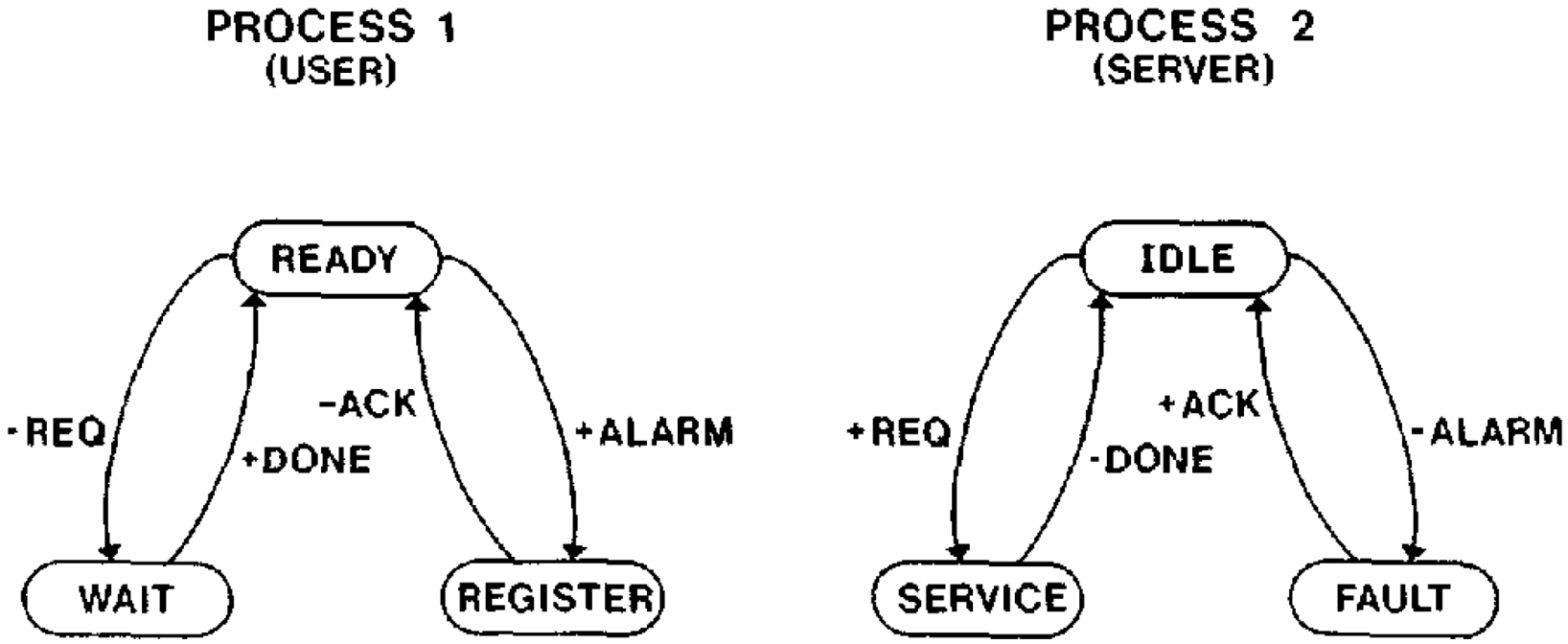
- An **operational** model for concurrent communicating processes

Communicating Finite-State Machines (CFSMs) [JACM83]

On Communicating Finite-State Machines

DANIEL BRAND AND PITRO ZAFIROPULO

IBM Zurich Research Laboratory, Ruschlikon, Switzerland



Process Algebraic Contracts [CAV04]

Stuck-Free Conformance

Cédric Fournet, Tony Hoare, Sriram K. Rajamani, and Jakob Rehof

Microsoft Research
{fournet,thoare,sriram,rehof}@microsoft.com

- **CCS** used to model behavioural contracts

$$C ::= \mathbf{1} \mid \sum_{i \in I} \alpha_i.C_i \mid X \mid recX.C$$

$$\alpha ::= a \mid \overline{a}$$

Basic Process Algebra (BPA)

$$\overline{\text{add\_to\_cart}}.\overline{\text{pay}}.\mathbf{1}$$

$$recX.\left(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1}\right)$$

- **CCS** used to model behavioural contracts

$$C \;::=\; \mathbf{1} \;\mid\; \textstyle\sum_{i\in I} \alpha_i.C_i \;\mid\; X \;\mid\; recX.C$$

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Basic Process Algebra (BPA)

$$\overline{\text{add\_to\_cart}}.\overline{\text{pay}}.\mathbf{1}$$

$$recX.\left(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1}\right)$$

- Operational semantics defined as a **labeled transition system** (LTS):

$$\frac{j \in I}{\sum_{i \in I} \alpha_i.C_i \xrightarrow{\alpha_j} C_j} \qquad \frac{C\{recX.C/X\} \xrightarrow{\lambda} C'}{recX.C \xrightarrow{\lambda} C'}$$

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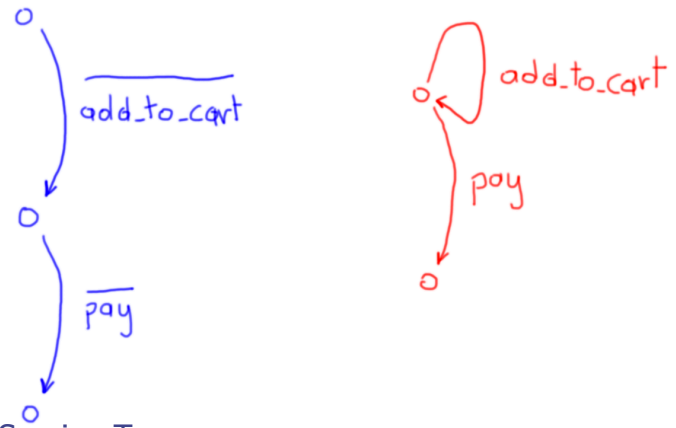


Coincidence between BPA and CFSMs

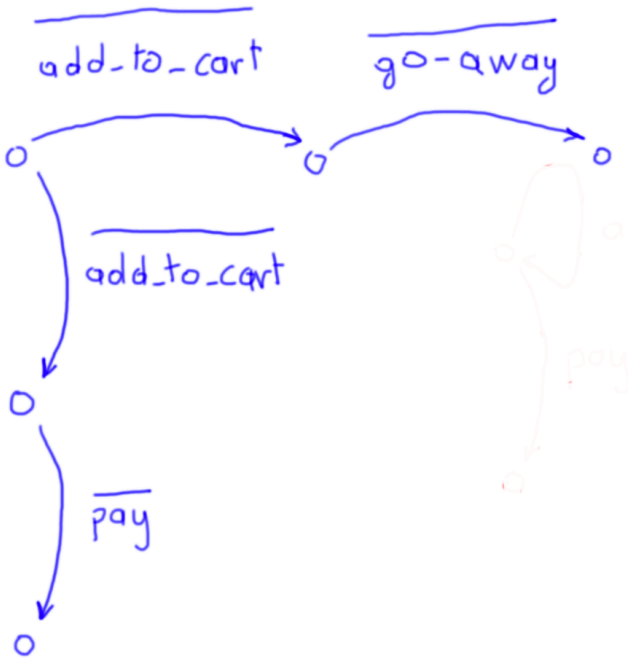
- The operational semantics of a **process algebraic contract** is a finite LTS, that can be interpreted as a **CFSM**
- Given a **CFSM**, it can be interpreted as a finite LTS, and it is possible to synthesize a **process algebraic contract** whose operational semantics is isomorphic with it

I have "cheated".. ;-)

- Actually, CFSMs are **deterministic** !

$$\overline{\text{add\_to\_cart}}.\overline{\text{pay}}.1 \;+\; \overline{\text{add\_to\_cart}}.\overline{\text{go\_away}}.1$$

Two transitions from the same state with the same label



Not admitted by CFSMs

Contract composition

- Behavioural contracts communicate by performing **complementary** actions
  - We use $C \| D$ to denote the (parallel) **composition** of two contracts
  - The **operational semantics** of contract compositions is given by a transition system:

$$\frac{C \xrightarrow{\overline{a}} C' \quad D \xrightarrow{a} D'}{C \| D \longrightarrow C' \| D'}$$

Contract composition

$$\overline{\text{add\_to\_cart}}.\overline{\text{pay}}.\mathbf{1} \| recX.\left(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1}\right)$$

$$\overline{\text{pay}}.\mathbf{1} \| recX.\left(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1}\right)$$

Successful completion

$$\mathbf{1} \| \mathbf{1}$$

$$\frac{C \xrightarrow{\overline{a}} C' \quad D \xrightarrow{a} D'}{C \| D \longrightarrow C' \| D'}$$

Contract compliance

- Two contracts are **compliant** when their composition is correct:
  - i.e. all possible computations
    - terminates with **successful completion**, or
    - **never** terminate

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- **Question time**: are

$$\overline{\text{add\_to\_cart}}.\overline{\text{pay}}.\mathbf{1} \;+\; \overline{\text{add\_to\_cart}}.\overline{\text{go\_away}}.\mathbf{1}$$

$$recX.\left(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1}\right)$$

compliant ?

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$$recX.\left(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1}\right)$$

compliant ?

NO !

Contract compliance

- Two contracts are **compliant** when their composition is correct:
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- **Question time (2)**: are

$$\overline{\text{add\_to\_cart}}.(\overline{\text{pay}}.\mathbf{1} + \overline{\text{go\_away}}.\mathbf{1})$$
$$recX.(\text{add\_to\_cart}.X + \text{pay}.\mathbf{1})$$

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compliant ?

YES !

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Contract refinement

- A contract C′ refines a contract C if it is a **compliance-preserving** substitute:

$$\forall D.\ \big(C \text{ is compliant with } D\big) \Rightarrow \big(C' \text{ is compliant with } D\big)$$

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NO !

Contract refinement ≠ Session subtyping

- We have that $\overline{\mathrm{add\_to\_cart}}.\overline{\mathrm{pay}}.\mathbf{1}$ is **not a refinement** of $recX.\left(\overline{\mathrm{add\_to\_cart}}.X + \overline{\mathrm{pay}}.\mathbf{1}\right)$
  - Consider the **discriminating** contract $\mathrm{pay}.\mathbf{1}$ which is compliant with the latter but not with the former
- .. while we know that the session type $\oplus\{\mathrm{add\_to\_cart} : \oplus\{\mathrm{pay} : \mathbf{end}\}\}$ is **subtype** of $\mu\mathbf{t}.\ \oplus\{\mathrm{add\_to\_cart} : \mathbf{t}, \mathrm{pay} : \mathbf{end}\}$

Observation

- In session types
  - output internal choices $\oplus$ are treated **differently** from input external choices $\&$
  - see e.g. output **covariance** and input **contravariance**
- While in BPA input and output are treated **specularly**
  - .. we need a **different operational model** for contracts !

Session contracts [MSCS14]

Math. Struct. in Comp. Science (2016), *vol.* 26, *pp.* 510–560.
doi:10.1017/S0960129514000243 First published online 10 November 2014

Modelling session types using contracts

GIOVANNI BERNARDI and MATTHEW HENNESSY†

- **No-mixed** deterministic choices

$$C \;::=\; \mathbf{1} \;\mid\; \sum_{i\in I} a^i.C_i \;\mid\; \sum_{i\in I} \overline{a^i}.C_i \;\mid\; X \;\mid\; recX.C$$

Session contracts [MSCS14]

$$\frac{j \in I}{\sum_{i \in I} a^i.C_i \xrightarrow{a^j} C_j} \qquad \frac{C\{recX.C/X\} \xrightarrow{\lambda} C'}{recX.C \xrightarrow{\lambda} C'}$$

$$\frac{j \in I}{\sum_{i \in I} \overline{a}^i.C_i \longrightarrow \left(\overline{a}^j.C_j\right) \xrightarrow{\overline{a}^j} C_j}$$

- **No-mixed** deterministic choices, with **output** selection

$$C \;::=\; \mathbf{1} \;\mid\; \sum_{i \in I} a^i.C_i \;\mid\; \sum_{i \in I} \overline{a^i}.C_i \;\mid\; X \;\mid\; recX.C$$

What about the counter-example?

- **Question time**: now that we restricted to session contracts, reconsider the two contracts

$$\overline{\text{add\_to\_cart}}.\overline{\text{pay}}.\mathbf{1}$$

$$recX.\left(\overline{\text{add\_to\_cart}}.X + \overline{\text{pay}}.\mathbf{1}\right)$$

  - is it possible to find a **discrimating** contract?
    - compatible with the latter and not with the former

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  - is it possible to find a **discrimating** contract?
    - compatible with the latter and not with the former

NO !

(Session) Contract refinement ≈ Session subtyping

- Natural **mapping** from session types to session contracts

$$[\![T = \oplus\{l_i : T_i\}_{i\in I}]\!] = \sum_{i\in I} \overline{l_i}.[\![T_i]\!]; \quad [\![T = \&\{l_i : T_i\}_{i\in I}]\!] = \sum_{i\in I} l_i.[\![T_i]\!]$$
$$[\![\mu \mathbf{t}.T]\!] = rec\,\mathbf{t}.[\![T]\!]; \quad [\![\mathbf{t}]\!] = \mathbf{t}; \quad [\![\mathbf{end}]\!] = \mathbf{1}.$$

- Theorem:
  - Given two session types T and S :
    T is a **subtype** of S if and only if
    $[[S]]$ is a session contract **refinement** of $[[T]]$

Conclusion for the synchronous case

- Session contracts give an **operational interpretation** to session types
- Session **subtyping** coincides with compliance-**refinement** for session contracts
- The Gay-Hole **algorithm** can be used to check contract refinement on session contracts

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Asynchronous Subtyping [ESOP09]

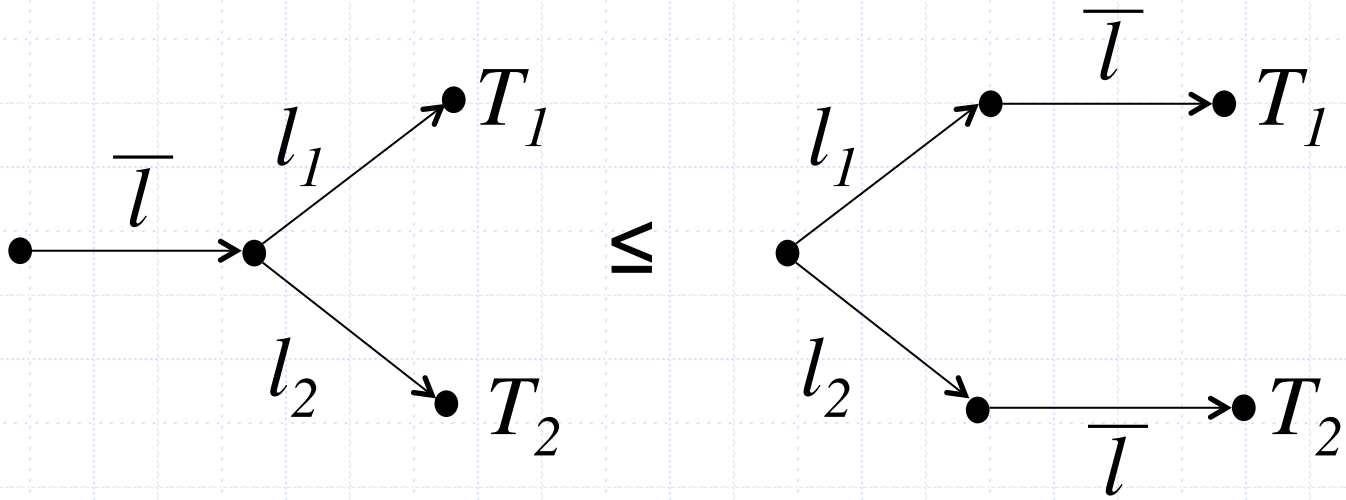
Global Principal Typing in Partially Commutative Asynchronous Sessions

Dimitris Mostrous[1], Nobuko Yoshida[1], and Kohei Honda[2]

- Besides output covariance / input contravariance a subtype may have outputs **anticipated** w.r.t. inputs

Asynchronous Subtyping [ESOP09]

$$\oplus\{l : \&\{l_1 : T_1, l_2 : T_2\}\} \leq \&\{l_1 : \oplus\{l : T_1\}, l_2 : \oplus\{l : T_2\}\}$$



- Besides output covariance / input contravariance a subtype may have outputs **anticipated** w.r.t. inputs

Synchronous Session Subtyping

Definition (Synchronous Subtyping, $\leq$). *$\mathcal{R}$ is a synchronous subtyping relation whenever $(T, S) \in \mathcal{R}$ implies that:*

1. *if $T = \mathbf{end}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \mathbf{end}$;*
2. *if $T = \oplus\{l_i : T_i\}_{i \in I}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \oplus\{l_j : S_j\}_{j \in J}$, $I \subseteq J$ and $\forall i \in I. (T_i, S_i) \in \mathcal{R}$;*
3. *if $T = \&\{l_i : T_i\}_{i \in I}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \&\{l_j : S_j\}_{j \in J}$, $J \subseteq I$ and $\forall j \in J. (T_j, S_j) \in \mathcal{R}$;*
4. *if $T = \mu \mathbf{t}.T'$ then $(T'\{T/\mathbf{t}\}, S) \in \mathcal{R}$.*

T is a synchronous subtype of S, written $T \leq S$, if there is a synchronous subtyping relation $\mathcal{R}$ such that $(T, S) \in \mathcal{R}$.

Input Context [I&C15]

Session typing and asynchronous subtyping for the higher-order π-calculus

Dimitris Mostrous [a,*], Nobuko Yoshida [b]

Definition (Input Context). *An input context $\mathcal{A}$ is a session type with multiple holes defined by the syntax:*

$$\mathcal{A} ::= \quad []^n \quad | \quad \&\{l_i : \mathcal{A}_i\}_{i \in I}$$

- Using input contexts:

$\mathcal{A}[T_k]^{k \in \{1,\ldots,m\}}$ *denotes the type obtained by filling each hole k in $\mathcal{A}$ with T_k*

Asynchronous Session Subtyping

Definition (Asynchronous Subtyping, $\leq$). *$\mathcal{R}$ is an asynchronous subtyping relation whenever $(T, S) \in \mathcal{R}$ implies that:*

1. *if $T = \mathbf{end}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \mathbf{end}$;*
2. *if $T = \oplus\{l_i : T_i\}_{i \in I}$ then $\exists n \geq 0, \mathcal{A}$ such that*
   - $\mathsf{unfold}^n(S) = \mathcal{A}[\oplus\{l_j : S_{kj}\}_{j \in J_k}]^{k \in \{1,\ldots,m\}}$,
   - $\forall k \in \{1, \ldots, m\}. I \subseteq J_k$ *and*
   - $\forall i \in I. (T_i, \mathcal{A}[S_{ki}]^{k \in \{1,\ldots,m\}}) \in \mathcal{R}$;
3. *if $T = \&\{l_i : T_i\}_{i \in I}$ then $\exists n \geq 0$ such that $\mathsf{unfold}^n(S) = \&\{l_j : S_j\}_{j \in J}$, $J \subseteq I$ and $\forall j \in J. (T_j, S_j) \in \mathcal{R}$;*
4. *if $T = \mu \mathbf{t}.T'$ then $(T'\{T/\mathbf{t}\}, S) \in \mathcal{R}$.*

T is an asynchronous subtype of S, written $T \leq S$, if there is an asynchronous subtyping relation $\mathcal{R}$ such that $(T, S) \in \mathcal{R}$.

Our previous example

$$\oplus\{l : \&\{l_1 : T_1, l_2 : T_2\}\} \leq \&\{l_1 : \oplus\{l : T_1\}, l_2 : \oplus\{l : T_2\}\}$$

- How to **prove** that these two types are actually in asynchronous subtyping relation?

Our previous example

$$\oplus\{l : \&\{l_1 : T_1, l_2 : T_2\}\} \leq \&\{l_1 : \oplus\{l : T_1\}, l_2 : \oplus\{l : T_2\}\}$$

- How to **prove** that these two types are actually in asynchronous subtyping relation?

$$\{ \quad (\oplus\{l : \&\{l_1 : T_1, l_2 : T_2\}\}, \&\{l_1 : \oplus\{l : T_1\}, l_2 : \oplus\{l : T_2\}\}) \ ,$$
$$(\&\{l_1 : T_1, l_2 : T_2\}\}, \&\{l_1 : T_1, l_2 : T_2\}\}) \ ,$$
$$\ldots \quad \}$$

  - the **input context** to be used is $\&\{l_1 : [\,]^1, l_2 : [\,]^2\}$ with the holes filled with $\oplus\{l : T_1\}$ and $\oplus\{l : T_2\}$, respectively

Question time

- Are

$$\oplus\{op : \&\{resp : \oplus\{huge\_data : \mathbf{end}\}\}\}$$

$$\oplus\{op : \oplus\{huge\_data : \&\{resp : \mathbf{end}\}\}\}$$

in **subtyping** relation?
Which is the **subtype**?

Question time

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  $\oplus\{op : \oplus\{huge\_data : \&\{resp : \mathbf{end}\}\}\}$

  in **subtyping** relation?

  Which is the **subtype**?

YES !

Question time

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$$\oplus\{op : \&\{resp : \oplus\{huge\_data : \mathbf{end}\}\}\}$$

$$\oplus\{op : \oplus\{huge\_data : \&\{resp : \mathbf{end}\}\}\}$$

in **subtyping** relation?

Which is the **subtype**?

YES !

The latter

Question time

- Are

$$\oplus\{op : \&\{resp : \oplus\{huge\_data : \mathbf{end}\}\}\}$$

$$\oplus\{op : \oplus\{huge\_data : \&\{resp : \mathbf{end}\}\}\}$$

Can we **automatize** this check?

A Sound Algorithm for Asynchronous Session Subtyping [CONCUR19]

Mario Bravetti
University of Bologna / INRIA FoCUS Team

Marco Carbone
IT University of Copenhagen

Julien Lange
University of Kent

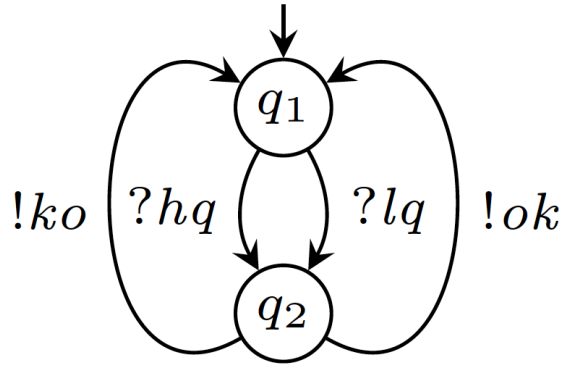
Nobuko Yoshida
Imperial College London

Gianluigi Zavattaro
University of Bologna / INRIA FoCUS Team

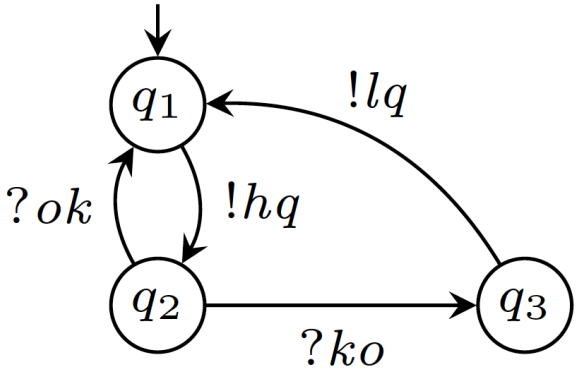
Demo time!

Question time

- Consider a **server** protocol:



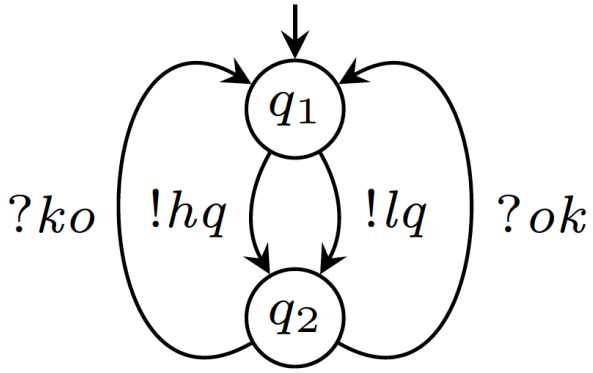
and a **client** protocol:
(! : outputs, ? : inputs)

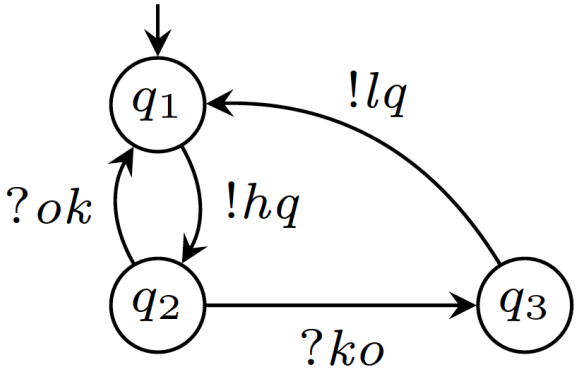


  - are they **compatible** considering asynchronous communication?

Solution

- Usual approach
  Consider the dual of
  the **server** protocol:





and check whether the
client protocol is
an **asynchronous** subtype
(we can try to use the CONCUR19 tool)

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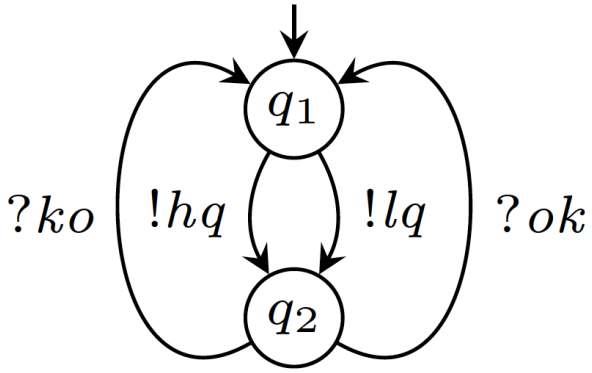

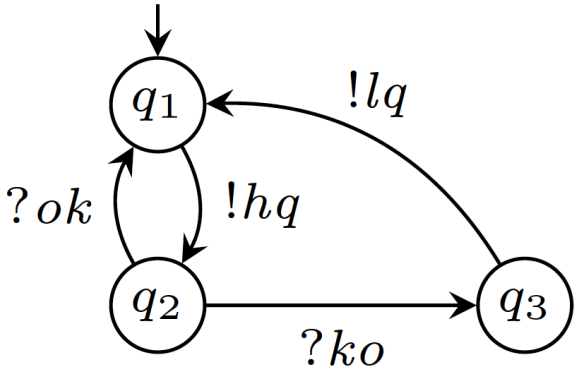

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Demo time!

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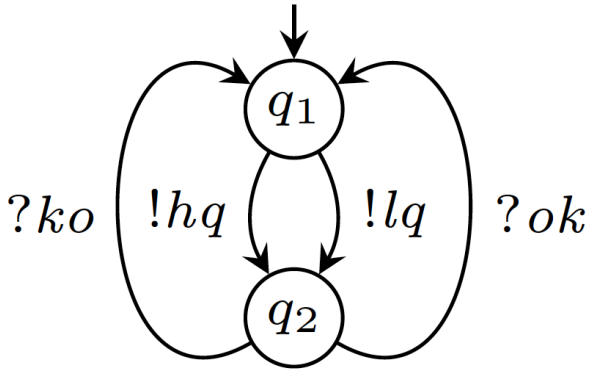
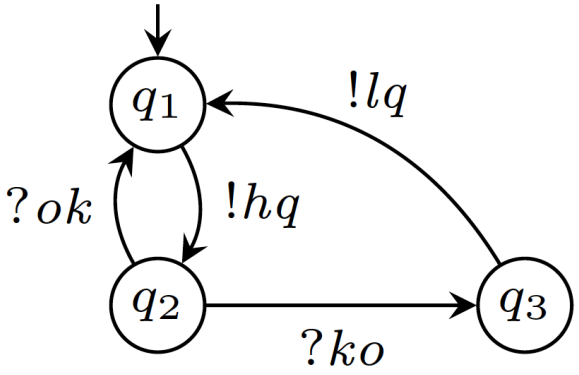




and check whether the client protocol is an **asynchronous** subtype

Demo time!

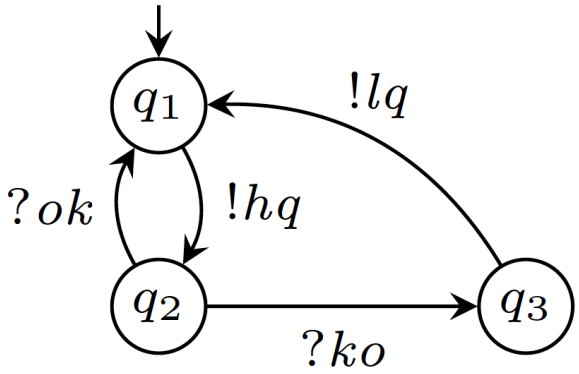
The tool allows us to reply positively !

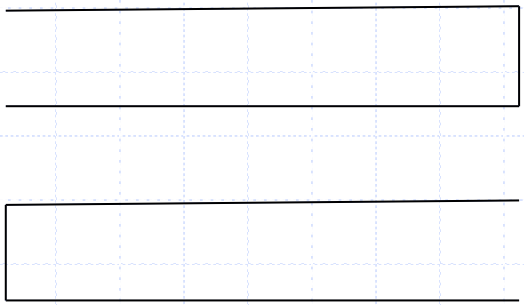
Structure of the course

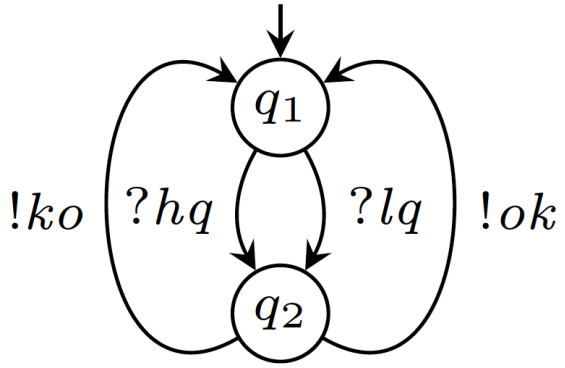
- Session types: subtyping
- Behavioural contracts: CFSMs, BPA
- The synchronous case
  - Session subtyping ≈ Contract refinement
- Asynchronous communication
  - Asynchronous session subtyping
  - Async. subtyping ≈ Async. refinement
  - Positive and Negative implications

Asynchronous contracts

- CFSMs communicate via **FIFO queues**







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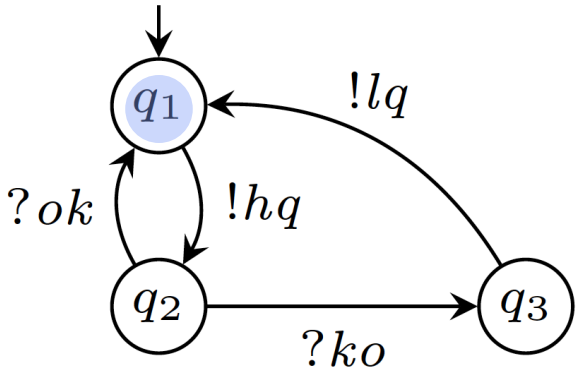

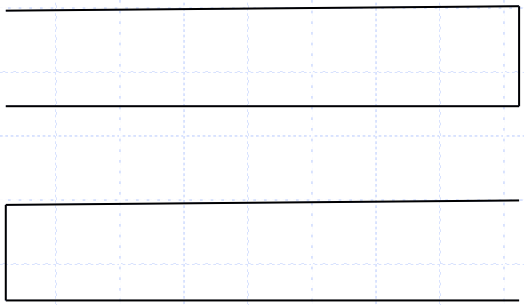

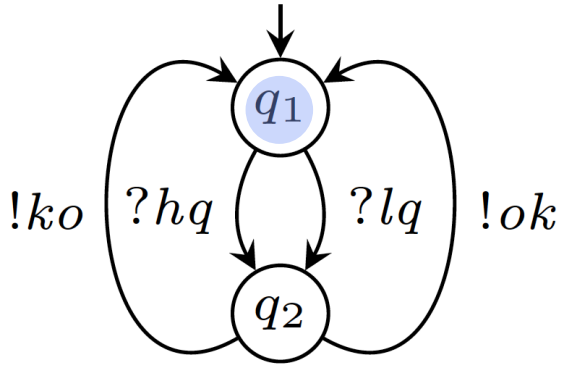

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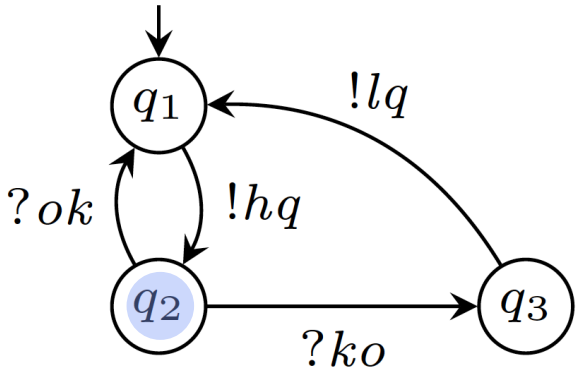

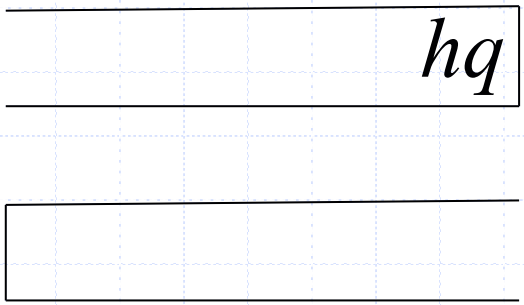

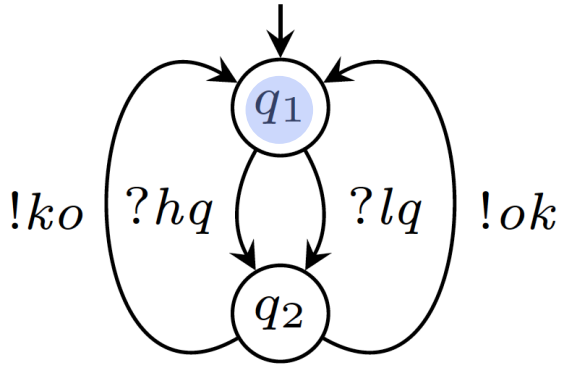

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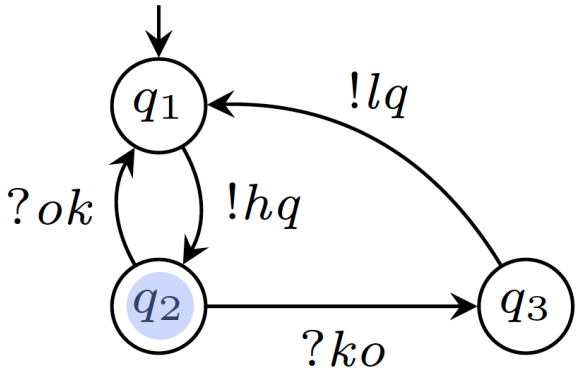

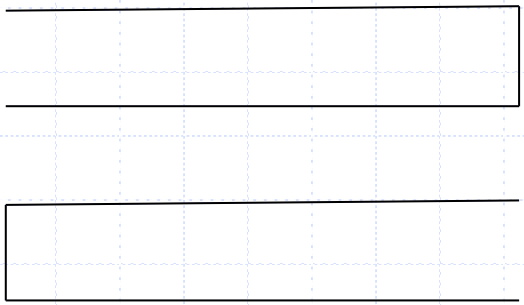

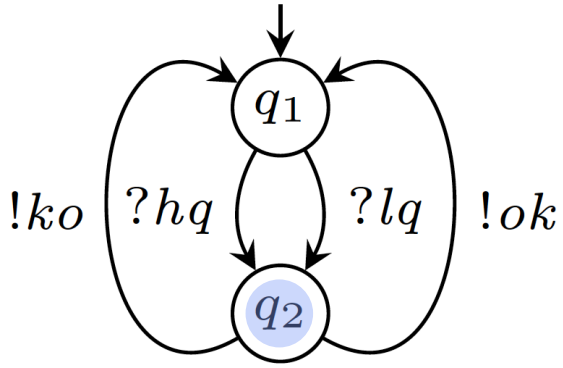

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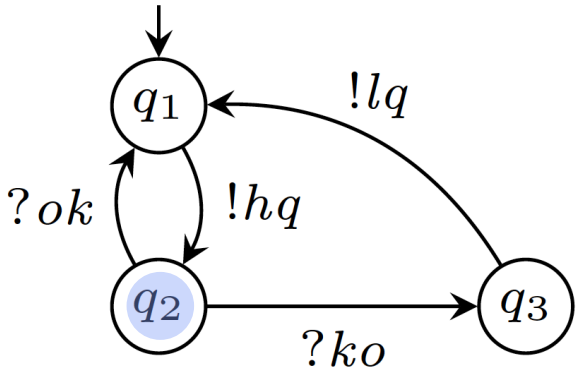

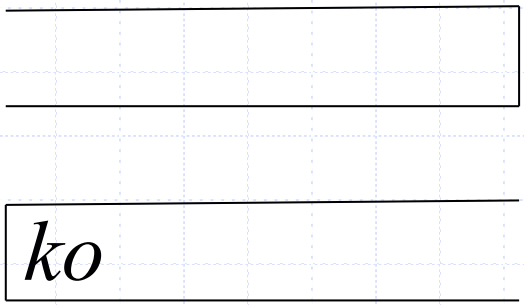

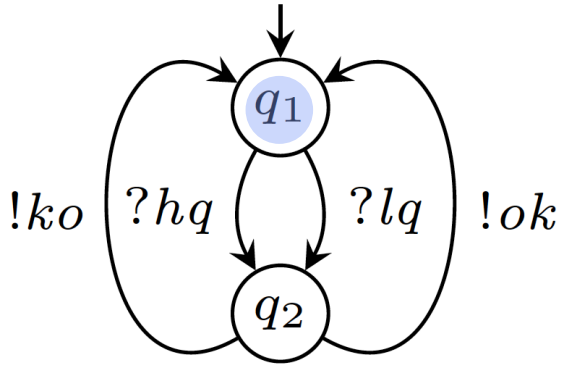

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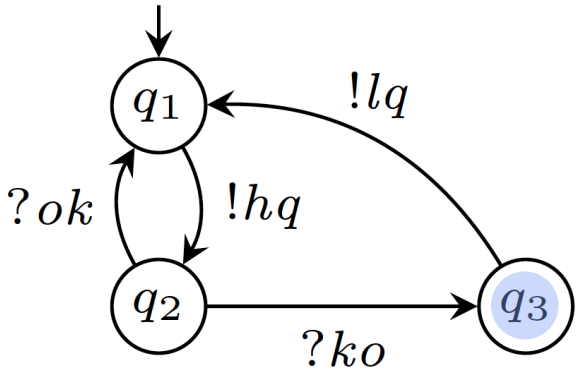

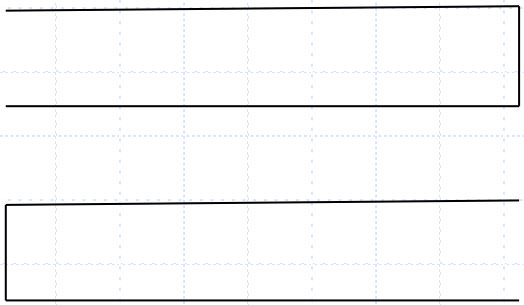

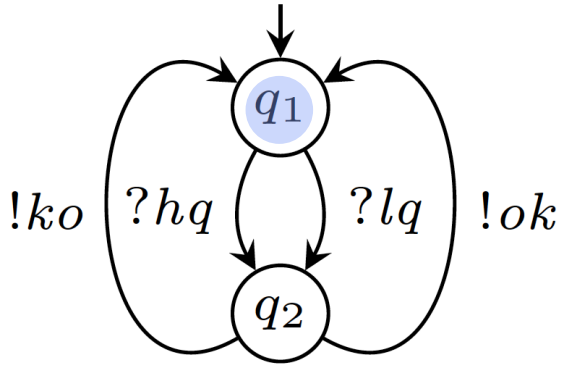

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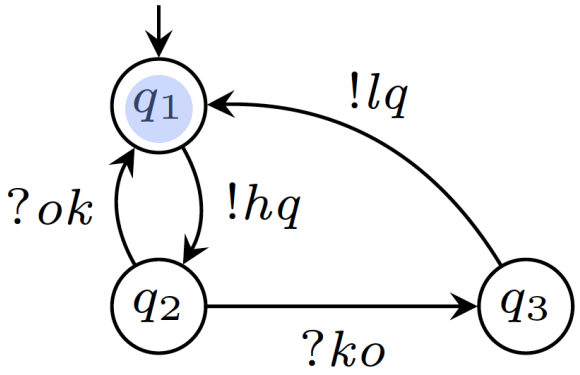

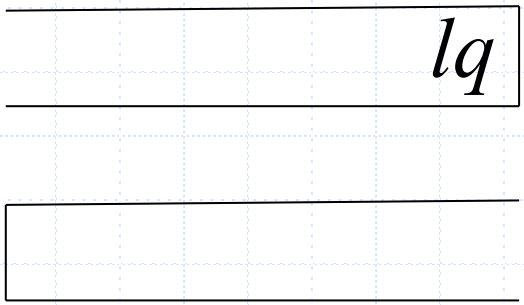

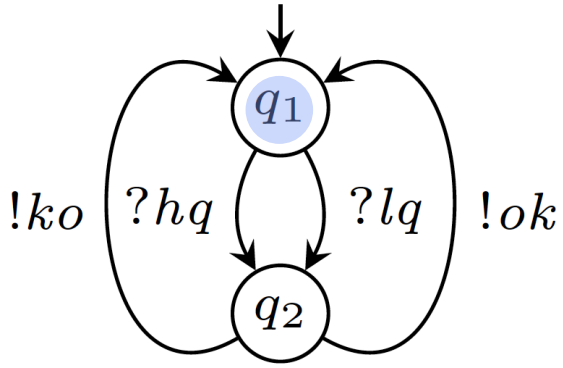

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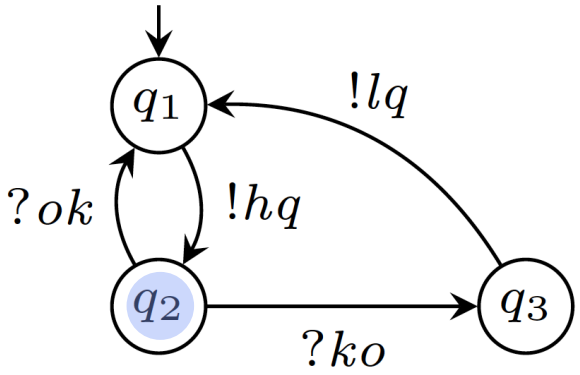

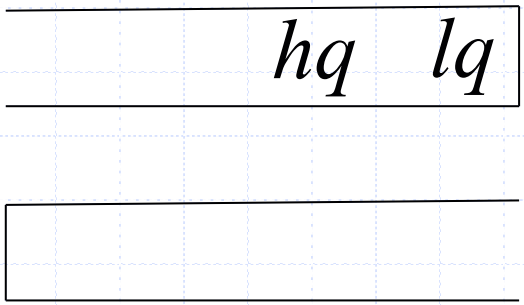

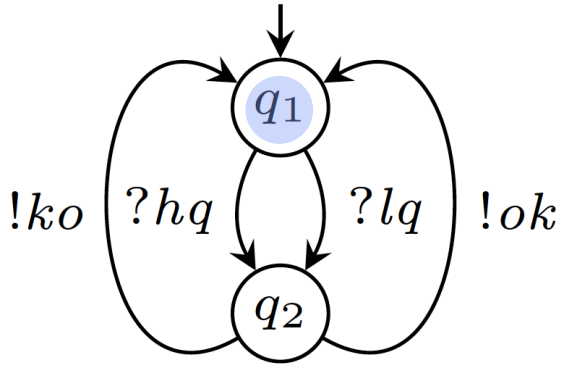

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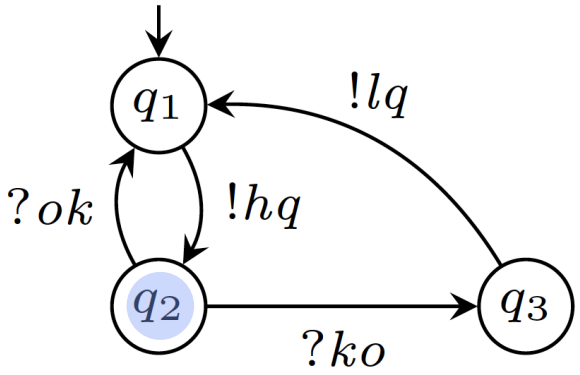

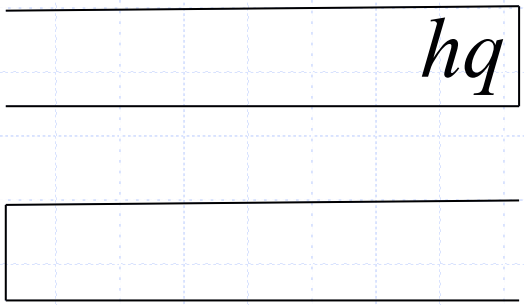

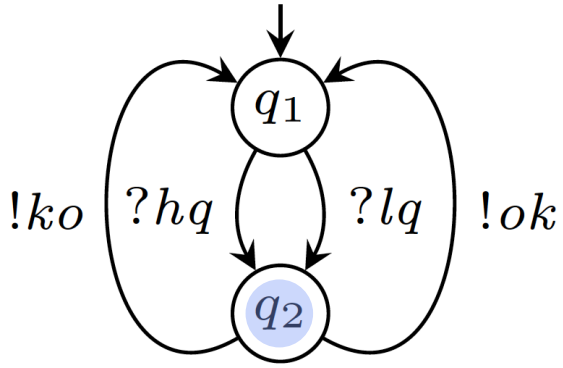

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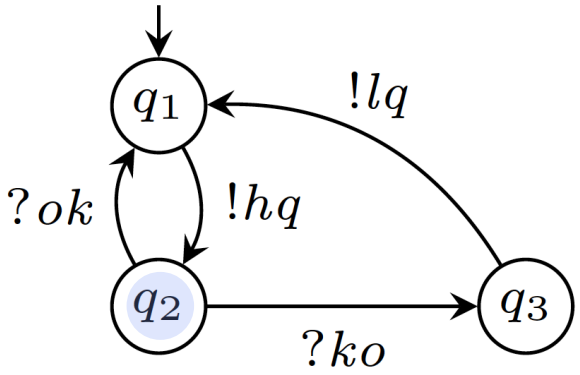
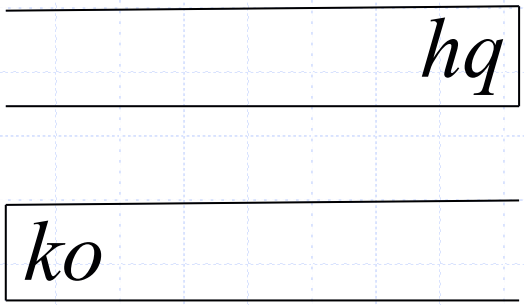
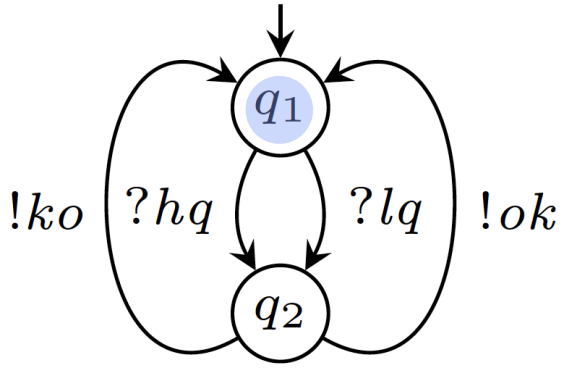
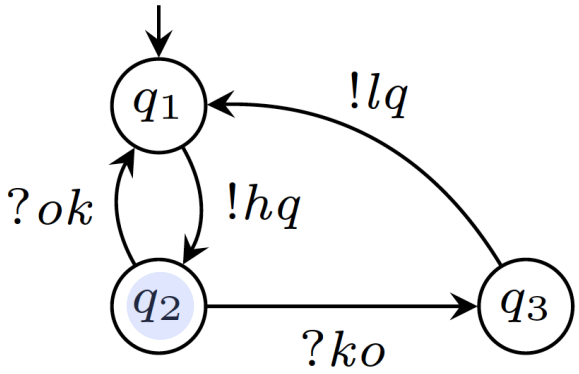


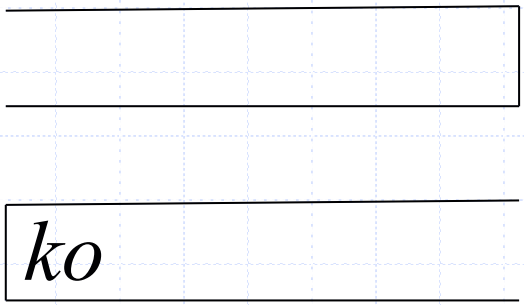


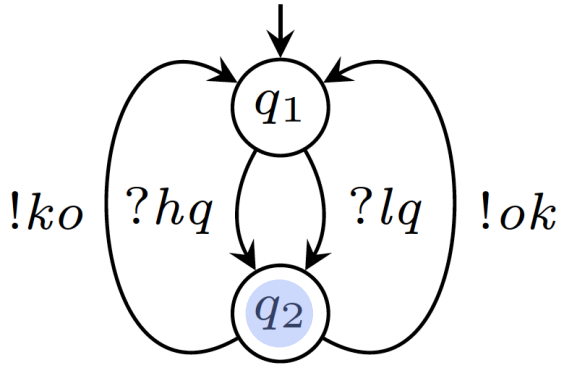


Asynchronous contracts

- CFSMs communicate via **FIFO queues**







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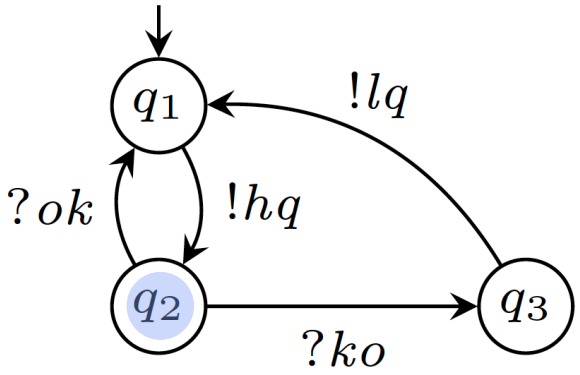

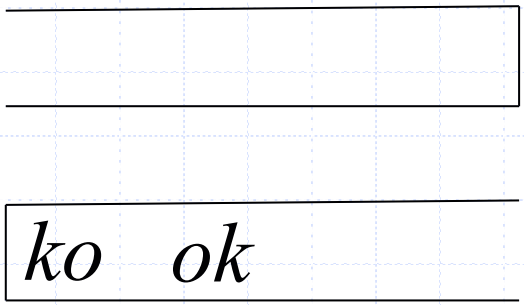

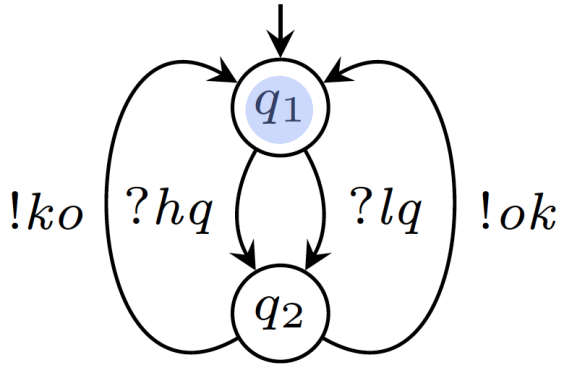

FIFO Process Algebra

- PA **enriched** with FIFO queues:

$$\begin{array}{rcl} P & ::= & [C, \mathcal{Q}] \| [D, \mathcal{Q}] \\ \mathcal{Q} & ::= & \epsilon \quad | \quad a :: \mathcal{Q} \end{array}$$

  - Rules for **consuming** messages

$$\frac{C \xrightarrow{a} C'}{[C, a :: \mathcal{Q}] \| [D, \mathcal{Q}'] \longrightarrow [C', \mathcal{Q}] \| [D, \mathcal{Q}']}$$

and for **sending** messages

$$\frac{C \xrightarrow{\overline{a}} C'}{[C, \mathcal{Q}] \| [D, \mathcal{Q}'] \longrightarrow [C', \mathcal{Q}] \| [D, \mathcal{Q}' :: a]}$$

Remember: Contract compliance

- Two contracts are **compliant** when their composition is correct:
  - i.e. all possible computations
    - terminates with **successful completion**, or
    - **never** terminate

Remember: Contract compliance

- Two contracts are **compliant** when their composition is correct:
  - i.e. all possible computations
    - terminates with **successful completion**, or
    - **never** terminate
- Re-define **composition** and **successful completion** for the asynchronous case:
  - Contract composition: $[C, \epsilon] \| [D, \epsilon]$
  - Successful completion: $[\mathbf{1}, \epsilon] \| [\mathbf{1}, \epsilon]$

Question time

- Are

$$recX.(\mathrm{but1}.\overline{\mathrm{coffe}}.X + \mathrm{but2}.\overline{\mathrm{tea}}.X)$$

$$recX.\overline{\mathrm{but1}}.\overline{\mathrm{but2}}.(\mathrm{coffe}.X + \mathrm{tea}.X)$$

compliant ?

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compliant ?

YES !

Remember: Contract refinement

- A contract C′ refines a contract C if it is a **compliance-preserving** substitute:

$$\forall D.\ (C \text{ is compliant with } D) \Rightarrow (C' \text{ is compliant with } D)$$

- This definition applies also to the **asynchronous** case

Question time

- Is $recX.\overline{\text{but1}}.\overline{\text{but2}}.(\text{coffe}.X + \text{tea}.X)$ a **refinement** of $recX.(\overline{\text{but1}}.\text{coffe}.X + \overline{\text{but2}}.\text{coffee}.X)$?
  - is it impossible to find a **discrimating** contract?
    - compatible with the latter and not with the former

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  - is it impossible to find a **discrimating** contract?
    - compatible with the latter and not with the former

YES !

Demo time!

Pay attention !

- We have used a (safe) algorithm for **session subtyping**, not an algorithm to check **contract refinement**!
  - But.. do they **coincide** also in the asynchronous case?

Async. contract refinement ≈ Async. session subtyping [SEFM19]

Relating Session Types and Behavioural Contracts: the Asynchronous Case

Mario Bravetti Gianluigi Zavattaro

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University of Bologna, Italy

- Additional **restriction** (besides session contracts):
  - Session types/contracts **cannot have infinite sequences of input**

Async. contract refinement ≈ Async. session subtyping [SEFM19]

- The type $\mu \mathbf{t}. \oplus \{l : \&\{l_1 : \mathbf{t}, l_2 : \mathbf{t}\}\}$ is **not** an asynchrononous subtype of $\mu \mathbf{t}.\&\{l_1 : \mathbf{t}, l_2 : \oplus\{l : \mathbf{t}\}\}$ because there exists **no input context** for the latter (infinite sequence of l_1 input choices)
- Additional **restriction** (besides session contracts):
  - Session types/contracts **cannot have infinite sequences of input**

Async. contract refinement ≈ Async. session subtyping [SEFM19]

- If we restrict to session types and session contracts **without infinite input** loops, we have the following ..
- Theorem:
  - Given two session types T and S: T is an **asynchronous subtype** of S if and only if $[[S]]$ is an **asynchronous** contract **refinement** of $[[T]]$

$$[\![T = \oplus\{l_i : T_i\}_{i\in I}]\!] = \sum_{i\in I} \overline{l_i}.[\![T_i]\!]; \quad [\![T = \&\{l_i : T_i\}_{i\in I}]\!] = \sum_{i\in I} l_i.[\![T_i]\!]$$
$$[\![\mu \mathbf{t}.T]\!] = rec\,\mathbf{t}.[\![T]\!]; \quad [\![\mathbf{t}]\!] = \mathbf{t}; \quad [\![\mathbf{end}]\!] = \mathbf{1}.$$

Structure of the course

- Session types: subtyping
- Behavioural contracts: CFSMs, BPA
- The synchronous case
  - Session subtyping ≈ Contract refinement
- Asynchronous communication
  - Asynchronous session subtyping
  - Async. subtyping ≈ Async. refinement
  - Positive and Negative implications

Positive implications

- Session contracts give an **operational interpretation** to session types also in the asynchronous case ..
  - .. under some additional restriction
  - Asynchronous session **subtyping** coincides with compliance-**refinement** for asynchronous session contracts
  - Async. session subtyping definition **can be used** to prove contract refinement

Negative implications [I&C17]

- Asynchronous session subtyping (hence also contract refinement) is **undecidable!**

Undecidability of asynchronous session subtyping

Mario Bravetti [a], Marco Carbone [b], Gianluigi Zavattaro [a,*]

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[b] *Department of Computer Science, IT University of Copenhagen, Rued Langgaards Vej 7, 2300 Copenhagen, Denmark*

- No general **algorithm** to check asynchronous session subtyping/contract refinement

Safe algorithmic characterization [CONCUR19]

A Sound Algorithm for Asynchronous Session Subtyping

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Marco Carbone
IT University of Copenhagen

Julien Lange
University of Kent

Nobuko Yoshida
Imperial College London

Gianluigi Zavattaro
University of Bologna / INRIA FoCUS Team

- Subtype check that replies with **True**, **False**, or **Maybe**

Demo time!

Algorithms for restricted cases [TCS18]

On the boundary between decidability and undecidability of asynchronous session subtyping

Mario Bravetti [a,*], Marco Carbone [b], Gianluigi Zavattaro [a]

[a] *University of Bologna, Department of Computer Science and Engineering/INRIA FOCUS, Italy*
[b] *Department of Computer Science, IT University of Copenhagen, Denmark*

- Algorithms for restricted cases:
  - **Bounded** buffers
  - **Single-out**: the output choices have always only one label
    - **No internal choices**, as in a reactive service

Related work

- **Orphan-message** free subtyping [LMCS17]
  - Messages are **eventually consumed**

ON THE PRECISENESS OF SUBTYPING IN SESSION TYPES*

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[b] Dip. di Informatica, Università di Torino, Corso Svizzera 185, 10149 Torino, Italy
e-mail address: dezani@di.unito.it

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  - Also this subtyping is **undecidable** [FOSSACS17]

On the Undecidability of Asynchronous Session Subtyping

Julien Lange(✉) and Nobuko Yoshida

Future work

- Design algorithms that **better** approximate asynchronous subtyping
- **Apply** asynchronous subtyping into session type tools/languages
- Consider the **multiparty** case:
  - **Synchronous** case recently studied

Precise subtyping for synchronous multiparty sessions [JLAMP19]

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[b] *Imperial College London, UK*
[c] *Høgskulen på Vestlandet, Norway*

Thank you!

- .. for your patience ;-)